## $(A G N)^{2}$

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## Selection Rules

- Selection Rules

- Allowed = Electric Dipole : Transitions which satisfy all the above selection rules are referred to as allowed transitions. These transitions are strong and have a typical lifetime of $\sim 10^{-8} \mathrm{~s}$. Allowed transitions are denoted without square brackets.

$$
\text { e.g., C IV } 1548,1550 \AA
$$

- Photons do not change spin, so transitions usually occur between terms with the same spin state ( $\Delta S=0$ ). However, relativistic effects mix spin states, particularly for high $Z$ atoms and ions. As a result, one can get (weak) spin changing transitions. These are called intercombination (semi-forbidden or intersystem) transitions or lines. They have a typical lifetime of $\sim 10^{-3} \mathrm{~s}$. An intercombination transition is denoted with a single right bracket.

$$
\mathrm{C}_{\text {III] }} 2 \mathrm{~s}^{2}{ }^{1} \mathrm{~S}-2 \mathrm{~s} 2 \mathrm{p}{ }^{3} \mathrm{P}^{\circ} \text { at } 1908.7 \AA . \quad(\Delta S=1)
$$

- If any one of the rules 1-4, 6-8 are violated, they are called forbidden transitions or lines. They have a typical lifetime of $\sim 1-10^{3} \mathrm{~s}$. A forbidden transition is denoted with two square brackets.

$$
1906.7 \AA[\mathrm{C} \operatorname{III}] 2 \mathrm{~s}^{2}{ }^{1} \mathrm{~S}_{0}-2 \mathrm{~s} 2 \mathrm{p}{ }^{3} \mathrm{P}_{2}^{\mathrm{o}}, \quad(\Delta S=1, \Delta J=2)
$$

- Resonance line denotes the longest wavelength, dipole-allowed transition arising from the ground state of a particular atom or ion.


## Forbidden Lines

- Forbidden lines are often difficult to study in the laboratory as collision-free conditions are needed to observe metastable states.
- In this context, it must be remembered that laboratory ultrahigh vacuums are significantly denser than so-called dense interstellar molecular clouds.
- Even in the best vacuum on Earth, frequent collisions knock the electrons out of these orbits (metastable states) before they have a chance to emit the forbidden lines.
- In astrophysics, low density environments are common. In these environments, the time between collisions is very long and an atom in an excited state has enough time to radiate even when it is metastable.
- Forbidden lines of nitrogen ([N II] at 654.8 and 658.4 nm ), sulfur ([S II] at 671.6 and 673.1 nm ), and oxygen ([ O II] at 372.7 nm , and [ O III] at 495.9 and 500.7 nm ) are commonly observed in astrophysical plasmas. These lines are important to the energy balance of planetary nebulae and H II regions.
- The forbidden 21-cm hydrogen line is particularly important for radio astronomy as it allows very cold neutral hydrogen gas to be seen.
- Since metastable states are rather common, forbidden transitions account for a significant percentage of the photons emitted by the ultra-low density gas in Universe.
- Forbidden lines can account for up to $90 \%$ of the total visual brightness of objects such as emission nebulae.


## History: Nebulium?

- In 1918, extensive studies of the emission spectra of nebulae found a series of lines which had not been observed in the laboratory.
- Particularly strong were features at $4959 \AA ̊$ and $5007 \AA ̊$. For a long time, this pair could not be identified and these lines were attributed to a new element, 'nebulium'.
- In 1927, Ira Bowen (1898-1973) discovered that the lines were not really due to a new chemical element but instead forbidden lines from doubly ionized oxygen [O III].
- He realized that in the diffuse conditions found in nebulae,


Optical spectra of NGC 6153, Liu et al. (2000, MNRAS) atoms and ions could survive a long time without undergoing collisions. Indeed, under typical nebula conditions the mean time between collisions is in the range $\mathbf{1 0 - 1 0 , 0 0 0}$ secs. This means that there is sufficient time for excited, metastable states to decay via weak, forbidden line emissions.

- The forbidden lines could not be observed in the laboratory where it was not possible to produce collision-free conditions over this long timeframe.
- Other 'nebulium' lines turned out to be forbidden lines originating from singly ionized oxygen [O II] and nitrogen [ N II].
[O III], [O II], [N II], etc: We use a pair of square brackets for a forbidden line.



## Notations

- Notations for Spectral Emission Lines and for Ions
- There is a considerable confusion about the difference between these two ways of referring to a spectrum or ion, for example, C III or $\mathrm{C}^{+2}$. These have very definite different physical meanings. However, in many cases, they are used interchangeably.
- $\mathrm{C}^{+2}$ is a baryon and C III is a set of photons.
- C+2 refers to carbon with two electrons removed, so that is doubly ionized, with a net charge of +2 .
- C III is the spectrum produced by carbon with two electrons removed. The C III spectrum will be produced by impact excitation of $\mathrm{C}^{+2}$ or by recombination of $\mathrm{C}^{+3}$. So, depending on how the spectrum is formed. C III may be emitted by $\mathrm{C}^{+2}$ or $\mathrm{C}^{+3}$.

$$
\begin{array}{ll}
\text { collisional excitation: } & C^{+2}+e^{-} \rightarrow C^{+2 *}+e^{-} \rightarrow C^{+2}+e^{-}+h \nu \\
\text { recombination: } & C^{+3}+e^{-} \rightarrow C^{+2}+h \nu
\end{array}
$$

- There is no ambiguity in absorption line studies - only C+2 can produce a C III absorption line. This had caused many people to think that C III refers to the matter rather than the spectrum.
- But this notation is ambiguous in the case of emission lines.
- Radio Recombination Lines
- Astronomers label each recombination line using the name of the element, the lower level number $n$, and successive letters in the Greek alphabet to denote the level change $\Delta n$.
- $\alpha$ for $\Delta n=1, \beta$ for $\Delta n=2, \gamma$ for $\Delta n=3$, etc.
- The recombination line produced by the transition between the $n=92$ and $n=91$ levels of a hydrogen atom is called the $\mathrm{H} 91 \alpha$ line.


## [Hydrogen Atom] : Fine Structure

- The discussion on H -atom levels has assumed that all states with the same principal quantum number, $n$, have the same energy.
- However, this is not correct: inclusion of relativistic (or magnetic) effects split these levels according to the total angular momentum quantum number $J$. The splitting is called fine structure.
- For hydrogen, $\quad S=\frac{1}{2} \rightarrow J=L \pm \frac{1}{2}$
- Spectroscopic notation: $\quad(2 S+1) L_{J}$

| configuration | L | S | J | term | level |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $n s$ | 0 | $1 / 2$ | 1/2 | ${ }^{2} S$ | ${ }^{2} S_{1 / 2}$ |
| $n p$ | 1 | $1 / 2$ | $1 / 2,3 / 2$ | ${ }^{2} P^{0}$ | ${ }^{2} P_{1 / 2}^{o},{ }^{2} P_{3 / 2}^{o}$ |
| $n d$ | 2 | $1 / 2$ | $3 / 2,5 / 2$ | ${ }^{2} D$ | ${ }^{2} D_{3 / 2},{ }^{2} D_{5 / 2}$ |
| $n f$ | 3 | $1 / 2$ | 5/2, 7/2 | ${ }^{2} F^{o}$ | ${ }^{2} F_{5 / 2}^{o},{ }^{2} F_{7 / 2}^{o}$ |

Note that the levels are called to be

$$
\text { singlet if } 2 S+1=1 \quad S=0, \quad J=L
$$

$$
\text { doublet if } 2 S+1=2 \quad S=1 / 2, \quad J=L \pm 1 / 2
$$

$$
\text { triplet if } 2 S+1=3 \quad S=1, J=L-1, L, L+1
$$ (when $\mathrm{L}>0$ )

- The above table shows the fine structure levels of the H atom.
- Note that the states with principal quantum number $n=2$ give rise to three fine-structure levels. In spectroscopic notation, these levels are $2^{2} S_{1 / 2}, 2^{2} P_{1 / 2}^{o}$ and $2^{2} P_{3 / 2}^{o}$.


## Hydrogen Atom : Hyperfine Structure

- Hyperfine Structure in the H atom

Coupling the nuclear spin $I$ to the total electron angular momentum $J$ gives the final angular momentum $F$. For hydrogen this means

$$
F=J+I=J \pm \frac{1}{2}
$$


[Kwok] Physics and Chemistry of the ISM [Bernath] Spectra of atoms and Molecules

## Hydrogen Atom : Allowed Transitions

## - Selection Rules

- Transitions are governed by selection rules which determine whether they can occur.
$\Delta n$ any
$\Delta l= \pm 1$

$\Delta S=0 \quad \longrightarrow$ For H atom, this is always satisfied as $S=1 / 2$ for all states.
$\Delta L=0, \pm 1($ not $L=0-0)$
$\Delta J=0, \pm 1(\operatorname{not} J=0-0)$


## For $\boldsymbol{H} \alpha$ transitions:



Not all $\mathrm{H} \alpha$ transitions which correspond to $n=2-3$ are allowed.

$$
\begin{aligned}
2 \mathrm{~s}_{\frac{1}{2}} & -3 \mathrm{p}_{\frac{1}{2}} \\
& \text { is allowed; } \\
& -3 \mathrm{p}_{\frac{3}{2}} \\
2 \mathrm{p}_{\frac{1}{2}}-3 \mathrm{~d}_{\frac{5}{2}} & \text { is not allowed; } \\
-3 \mathrm{~s}_{\frac{1}{2}} & \text { is allowed; }(\Delta J=2) \\
& -3 \mathrm{~d}_{\frac{3}{2}}
\end{aligned} \text { is allowed; } \quad \begin{aligned}
2 \mathrm{p}_{\frac{3}{2}}-3 \mathrm{~s}_{\frac{1}{2}} & \text { is allowed; } \\
-3 \mathrm{~d}_{\frac{3}{2}} & \text { is allowed; } \\
-3 \mathrm{~d}_{\frac{5}{2}} & \text { is allowed. }
\end{aligned}
$$

The transition between $2 s-1 s$ is not allowed $(\Delta l=0)$.

- Hydrogen: lifetime of excited states
$\tau_{i}=\left(\sum_{j} A_{i j}\right)^{-1}$
where $A_{i j}$ is the Einstein A coefficient

| Level | 2 s | 2 p | 3 s | 3 p | 3 d |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\tau / \mathrm{s}$ | 0.14 | $1.6 \times 10^{-9}$ | $1.6 \times 10^{-7}$ | $5.4 \times 10^{-9}$ | $2.3 \times 10^{-7}$ |

- Lifetimes for allowed transitions are short, a few times $10^{-9} \mathrm{~s}$.
- However, the lifetime for the (2s) $2^{2} S_{1 / 2}$ level is $\sim 0.14 \mathrm{~s}$, which is $10^{8}$ times longer than the $2 p$ states. (The level is called to be metastable.)


## - Two-photon continuum radiation

- In low-density environments (e.g., ISM), an electron in the $2^{2} S_{1 / 2}$ level can jumps to a virtual $p$ state, which lies between $n=1$ and $n=2$ levels. The electron then jumps from this virtual state to the ground state, in the process emitting two photons with total frequency $\nu_{1}+\nu_{2}=\nu_{\mathrm{Ly} \alpha}$.
- Since this virtual $p$ state can occur anywhere between $n=1$ and $n=2$, continuum emission longward of Ly $\alpha$ will result.
- Because the radiative lifetime of the 2 s level is long. we need to consider the possibility for collisions with electrons and protons to depopulate 2 s level before a spontaneous decay occurs. However, the critical density, at which deexcitation by electron and proton collision is equal to the radiative decay rate, is $n_{\text {crit }} \approx 1880 \mathrm{~cm}^{-3}$. In the ISM, the radiative decay is in general faster than the collisional depopulation process.



| Element | $\mathrm{I} \rightarrow \mathrm{II}$ | $\mathrm{II} \rightarrow \mathrm{III}$ | $\mathrm{III} \rightarrow \mathrm{IV}$ | $\mathrm{IV} \rightarrow \mathrm{V}$ | $\mathrm{V} \rightarrow \mathrm{VI}$ | $\mathrm{VI} \rightarrow$ VII | $\mathrm{VII} \rightarrow$ VIII |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 H | 13.5984 |  |  |  |  |  |  |
| 2 He | 24.5874 | 54.416 |  |  |  |  |  |
| 3 Li | 5.3917 | 75.640 | 122.454 |  |  |  |  |
| 4 Be | 9.3227 | 18.211 | 153.894 | 217.719 |  |  |  |
| 5 B | 8.2980 | 25.155 | 37.931 | 259.375 | 340.226 |  |  |
| 6 C | 11.2603 | 24.383 | 47.888 | 64.494 | 392.089 | 489.993 |  |
| 7 N | 14.5341 | 29.601 | 47.449 | 77.474 | 97.890 | 552.072 | 667.046 |
| 8 O | 13.6181 | 35.121 | 54.936 | 77.414 | 113.899 | 138.120 | 739.293 |
| 9 F | 17.4228 | 34.971 | 62.708 | 87.140 | 114.243 | 147.163 | 185.189 |
| 10 Ne | 21.5645 | 40.963 | 63.423 | 97.117 | 126.247 | 154.214 | 207.271 |
| 11 Na | 5.1391 | 47.286 | 71.620 | 98.91 | 138.40 | 172.183 | 208.50 |
| 12 Mg | 7.6462 | 15.035 | 80.144 | 109.265 | 141.270 | 186.76 | 225.02 |
| 13 Al | 5.9858 | 18.829 | 28.448 | 119.992 | 153.825 | 190.477 | 241.76 |
| 14 Si | 8.1517 | 16.346 | 33.493 | 45.142 | 166.767 | 205.267 | 246.481 |
| 15 P | 10.4867 | 19.769 | 30.203 | 51.444 | 65.025 | 220.422 | 263.57 |
| 16 S | 10.3600 | 23.338 | 34.790 | 47.222 | 72.594 | 88.053 | 280.948 |
| 17 Cl | 12.9676 | 23.814 | 39.911 | 53.465 | 67.819 | 97.030 | 114.201 |
| 18 Ar | 15.7596 | 27.630 | 40.735 | 59.686 | 75.134 | 91.00 | 124.328 |
| 19 K | 4.3407 | 31.628 | 45.806 | 60.913 | 82.66 | 99.4 | 117.6 |
| 20 Ca | 6.1132 | 11.872 | 50.913 | 67.27 | 84.51 | 108.8 | 127.2 |
| 21 Sc | 6.5615 | 12.800 | 24.757 | 73.489 | 91.69 | 110.7 | 138.0 |
| 22 Ti | 6.8281 | 13.576 | 24.492 | 43.267 | 123.7 | 119.533 | 140.846 |
| 23 V | 6.7462 | 14.655 | 29.311 | 46.709 | 65.282 | 128.125 | 150.641 |
| 24 Cr | 6.7665 | 16.486 | 30.959 | 49.160 | 69.456 | 90.635 | 160.175 |
| 25 Mn | 7.4340 | 15.640 | 33.668 | 51.2 | 72.4 | 95.60 | 119.203 |
| 26 Fe | 7.9024 | 16.188 | 30.651 | 54.801 | 75.010 | 99.063 | 124.976 |
| 27 Co | 7.8810 | 17.084 | 33.50 | 51.27 | 79.5 | 102. | 129. |
| 28 Ni | 7.6398 | 18.169 | 35.187 | 54.925 | 76.06 | 107.87 | 133. |
| 29 Cu | 7.7264 | 20.292 | 36.841 | 57.380 | 79.846 | 103.031 | 138.862 |
| 30 Zn | 9.3492 | 17.964 | 39.723 | 59.573 | 82.574 | 133.903 | 133.903 |

[Draine] Physics of the Interstellar and Intergalactic Medium

## Notes:

- Ionization potentials from Ralchenko et al. (2010)
- The light line separates ions with $I<I_{\mathrm{He}}$ from ions with $I>I_{\mathrm{He}}=24.6 \mathrm{eV}$.
- Ions to right of the heavy line (with $I>I_{\mathrm{He} \mathrm{II}}=54.4 \mathrm{eV}$ ) are not abundant in gas photoionized by O or B stars and are therefore indicative of photoionization by WR stars, PN nuclei, or collisional ionization in shocked gas.


## Energy Level Diagrams

- 1 electron

[Draine] Physics of the Interstellar and Intergalactic Medium


## - 3 electrons (Lithium-like ions)

-     - $(13.6 \mathrm{eV}) / \mathrm{hc}=109692 \mathrm{~cm}^{-1}$

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