# Interstellar Medium (ISM)

Week 7
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# H I Spin Temperature

- ***Collionsional rate coefficients***:
  - Collision with other H atoms

(Allison & Dalgarno 1969; Zygelman 2005)

$$k_{10}(\mathrm{H}) \approx \begin{cases} 1.19\times 10^{-10} T_2^{0.74-0.20\ln T_2}\ \mathrm{cm^3\,s^{-1}} & (20\,\mathrm{K} < T < 300\,\mathrm{K}) \\ 2.24\times 10^{-10} T_2^{0.207} \mathrm{e}^{-0.876/T_2}\ \mathrm{cm^3\,s^{-1}} & (300\,\mathrm{K} < T < 10^3\,\mathrm{K}) \end{cases}$$

$$k_{01}(\mathrm{H}) \approx 3k_{10}(\mathrm{H})e^{-0.0682\,\mathrm{K}/T}$$

$$T_2 \equiv T/100\,\mathrm{K}$$

$$\boxed{k_{\ell u} = \frac{\mathrm{g}_u}{\mathrm{g}_\ell} k_{u\ell} \exp\left(-\frac{E_{u\ell}}{k_\mathrm{B} T}\right)}$$

  - Collision with electrons

(Furlanetto & Furlanetto 2007)

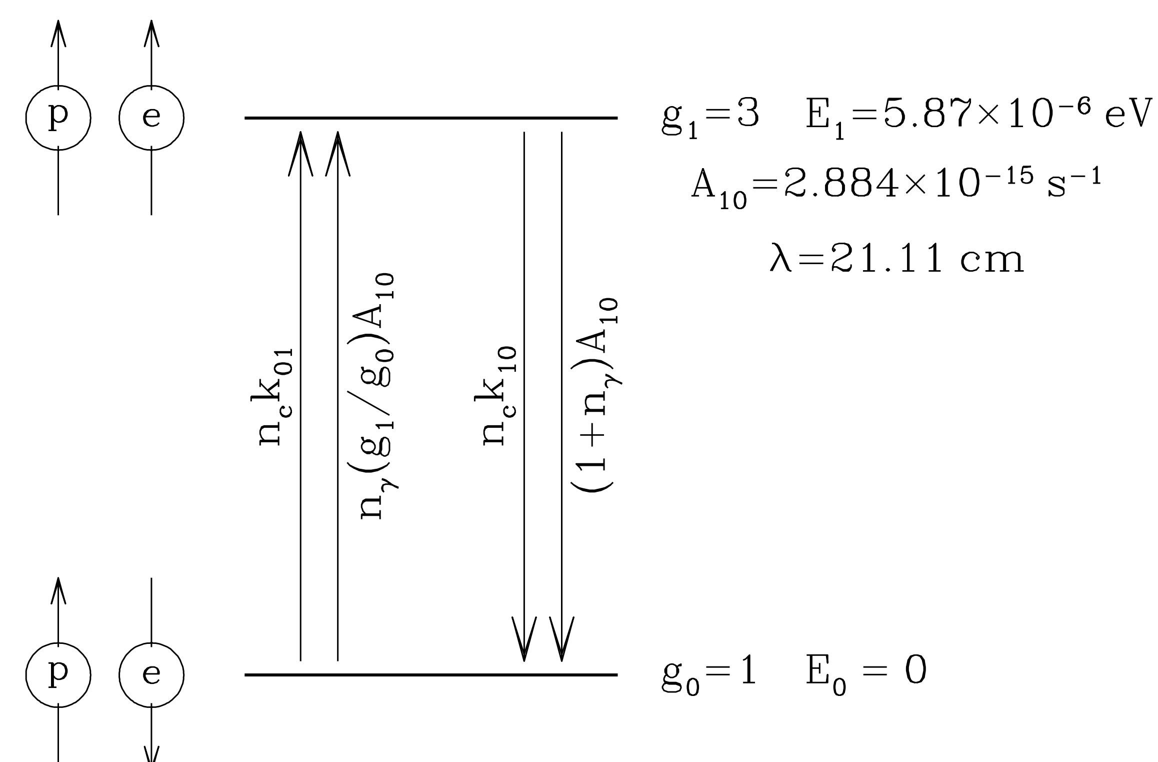
$$k_{10}(e^-) \approx 2.26\times 10^{-9}\,(T/100\,\mathrm{K})^{0.5}\ \mathrm{cm^3\,s^{-1}} \quad (1 \lesssim T \lesssim 500\,\mathrm{K})$$

$$k_{01}(e^-) \approx 3k_{10}(e^-)e^{-0.0682\,\mathrm{K}/T}$$

- This is a factor ~ 10 larger than that for H atoms. However, ***electrons will be minor importance in regions with a fractional ionization*** $x_e \lesssim 0.03$, such as the CNM and WNM.

excitation (spin) temperature vs. kinetic temperature

$$\frac{n_1}{n_0} = \frac{n_c k_{01} + \bar{n}_\gamma (\mathrm{g}_1/\mathrm{g}_0) A_{10}}{n_c k_{10} + (1+\bar{n}_\gamma) A_{10}} \qquad \begin{aligned} k_{10} &= k_{10}(H) + k_{10}(e^-) \\ k_{01} &= k_{01}(H) + k_{01}(e^-) \end{aligned}$$




[Figure 17.1 in Draine]

- Radiation Field strength
  - The radiation field near 21 cm is dominated by the cosmic microwave background plus Galactic synchrotron emission. The antenna temperature is

$$T_A \approx T_{\rm CMB} + T_{\rm syn} = 2.73\,{\rm K} + 1.04\,{\rm K} = 3.77\,{\rm K}$$

  - Photon occupation number:

$$\bar{n}_\gamma = \left(e^{h\nu/kT_{\rm rad}} - 1\right)^{-1} \approx \frac{kT_A}{h\nu} \approx \frac{3.77\,{\rm K}}{0.0682\,{\rm K}} \approx 55$$

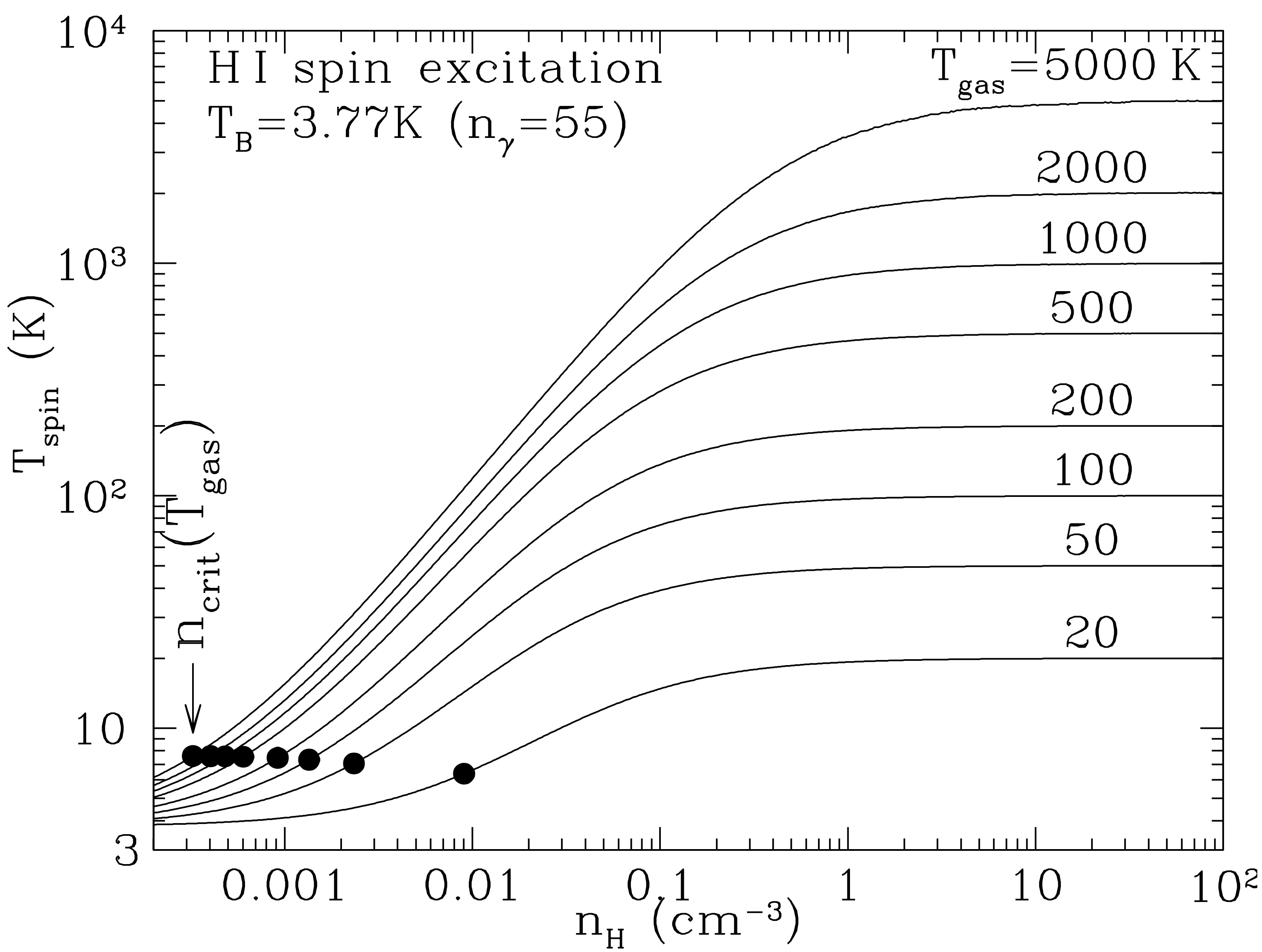
  - The critical density is then

$$\begin{aligned} n_{\rm crit}(H) &= \frac{(1+\bar{n}_\gamma)A_{10}}{k_{10}} \\ &\approx 0.02\,{\rm cm}^{-3} \qquad\qquad \text{at } T \sim 10\,{\rm K} \\ &\approx 1.4\times 10^{-3}\,{\rm cm}^{-3} \quad \text{at } T \sim 100\,{\rm K} \\ &\approx 5\times 10^{-4}\,{\rm cm}^{-3} \quad\;\; \text{at } T \sim 1000\,{\rm K} \end{aligned}$$

- H I spin temperature as a function of density $n_{\mathrm{H}}$, including only 21 cm continuum radiation and collisions with H atoms. Ly$\alpha$ scattering is not included.
  - Filled circles show $n_{\mathrm{crit}}(\mathrm{H})$ for each temperature.
  - It is important to note that one requires $n \gg n_{\mathrm{crit}}$ in order to have $T_{\mathrm{spin}}$ within, say, 10% of $T_{\mathrm{gas}}$, particularly at high temperatures.

$$\frac{n_u}{n_\ell} = \frac{n_c k_{\ell u} + \bar{n}_\gamma (g_u/g_\ell) A_{u\ell}}{n_c k_{u\ell} + (1+\bar{n}_\gamma) A_{u\ell}}$$

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-E_{u\ell}/k_{\mathrm{B}} T_{\mathrm{exc}}\right)$$




[Fig. 17.2 in Draine]

Note that Ryden states that "in the CNM and WNM, we expect the hyperfine levels of atomic hydrogen to be collisionally excited, and to have a spin temperature close to the gas temperature." based on that $n_{\mathrm{crit}} \sim 6\times10^{-4}$ cm$^{-3}$ at T ~ 1000 K.

***The collisional excitation is strong enough, only in the CNM, to bring the spin temperature close to the gas kinetic temperature.***

However, this is not true in the WNM. In the WNM, the WF effect can thermalize the 21-cm spin temperature to the gas kinetic temperature.

***In the CNM, the 21-cm spin temperature is a good tracer of the gas kinetic temperature.*** *However, this is not true for other levels in other atoms.*

# C II Fine Structure Excitation

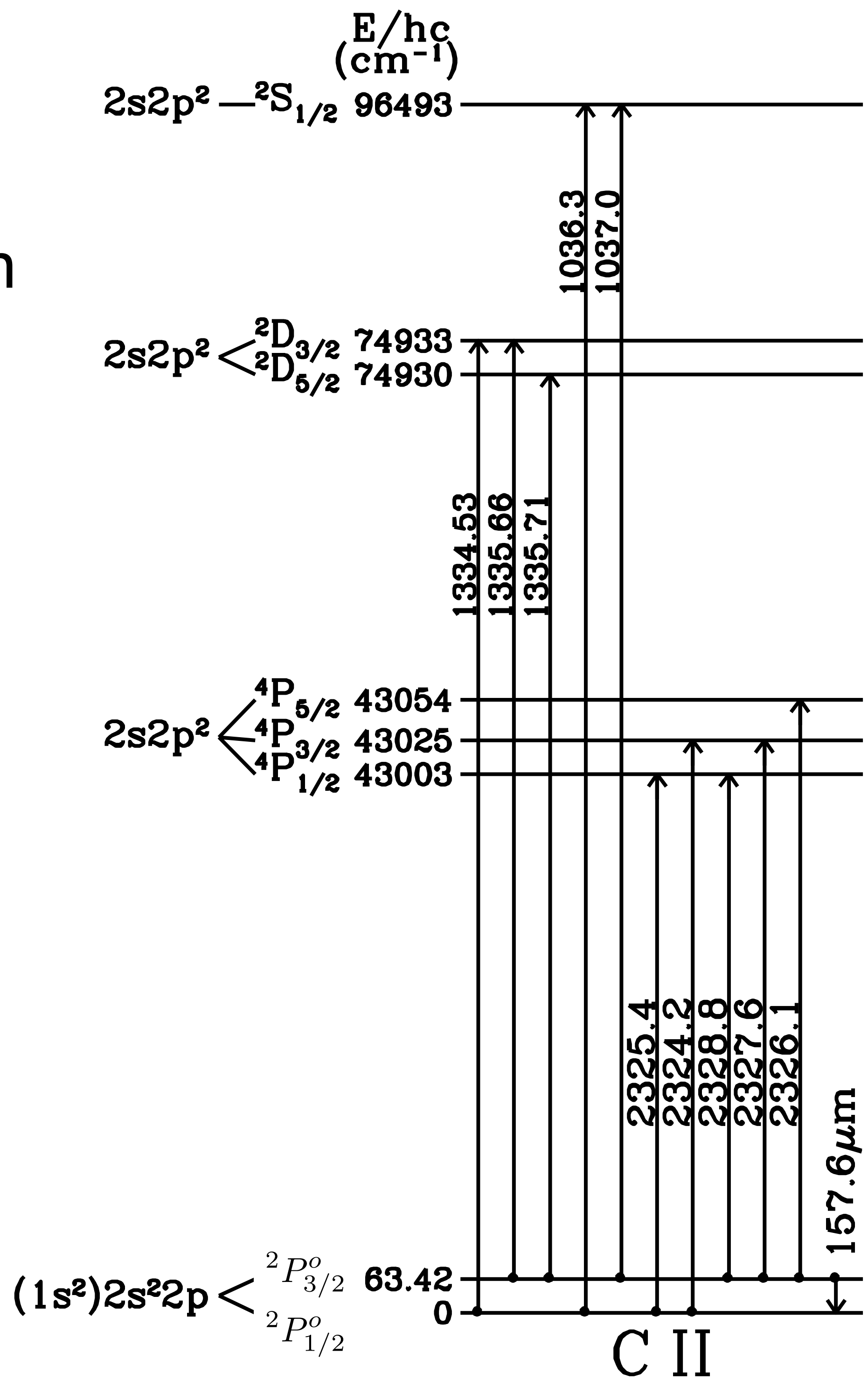
- The ground electronic state $1s^2 2s^2 2p\ {}^2P^o$ of $C^+$ contains two fine-structure levels.

- The electronically excited states have an excitation energy that is much higher than the kinetic temperature of the CNM.

$$2235\,\text{Å} \rightarrow E_{u\ell} = 0.56\,\text{eV} \rightarrow T = 6440\,\text{K}$$

- We may, therefore, consider the two fine-structure levels in the ground electronic state to be a two level atom.

- Will the populations of these two levels be thermalized in the ISM?



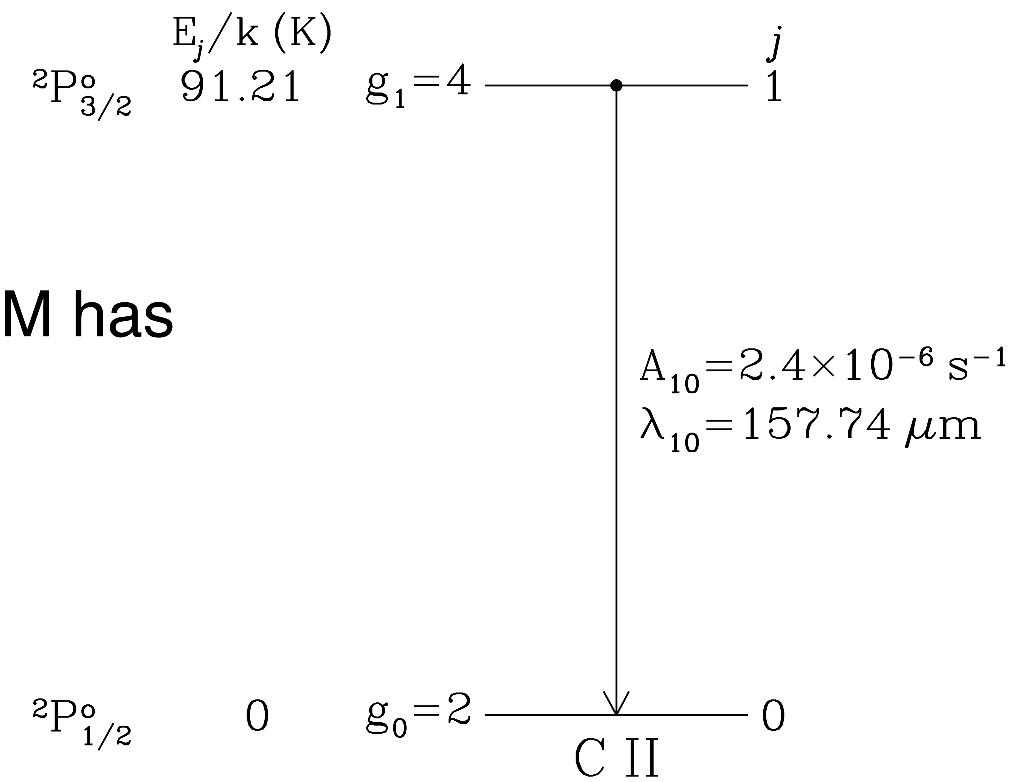
- Rate coefficients for collisional de-excitation:

$$\left\langle \Omega\left({}^2P^o_{1/2},\, {}^2P^o_{3/2}\right)\right\rangle \approx 2.1 \qquad\qquad \left(T_4 = T/10^4\,\mathrm{K},\; T_2 = T/10^2\,\mathrm{K}\right)$$

$$k_{10}(e^-) \approx 4.53\times 10^{-8}\, T_4^{-1/2}\,\mathrm{cm^3\,s^{-1}}$$

$$k_{10}(\mathrm{H}) \approx 7.58\times 10^{-10}\, T_2^{0.1281+0.0087\ln T_2}\,\mathrm{cm^3\,s^{-1}}$$

(Barinovs et al. 2005)




[Figure 17.3 in Draine]

- At $\lambda = 158\,\mu\mathrm{m}$, the continuum background in the ISM has

$$\bar{n}_\gamma \approx 10^{-5} \ll 1 \longrightarrow n_{\rm crit} \simeq \frac{A_{10}}{k_{10}}$$

- Critical densities:

$$n_{\rm crit}(e^-) \approx 53\, T_4^{1/2}\,\mathrm{cm^{-3}}$$

$$n_{\rm crit}(\mathrm{H}) \approx 3.2\times 10^3\, T_2^{-0.1281-0.0087\ln T_2}\,\mathrm{cm^{-3}}$$

- The critical densities are much higher than the typical densities in both the CNM and WNM. Thus, **the C II fine-structure levels will be sub-thermally excited.**

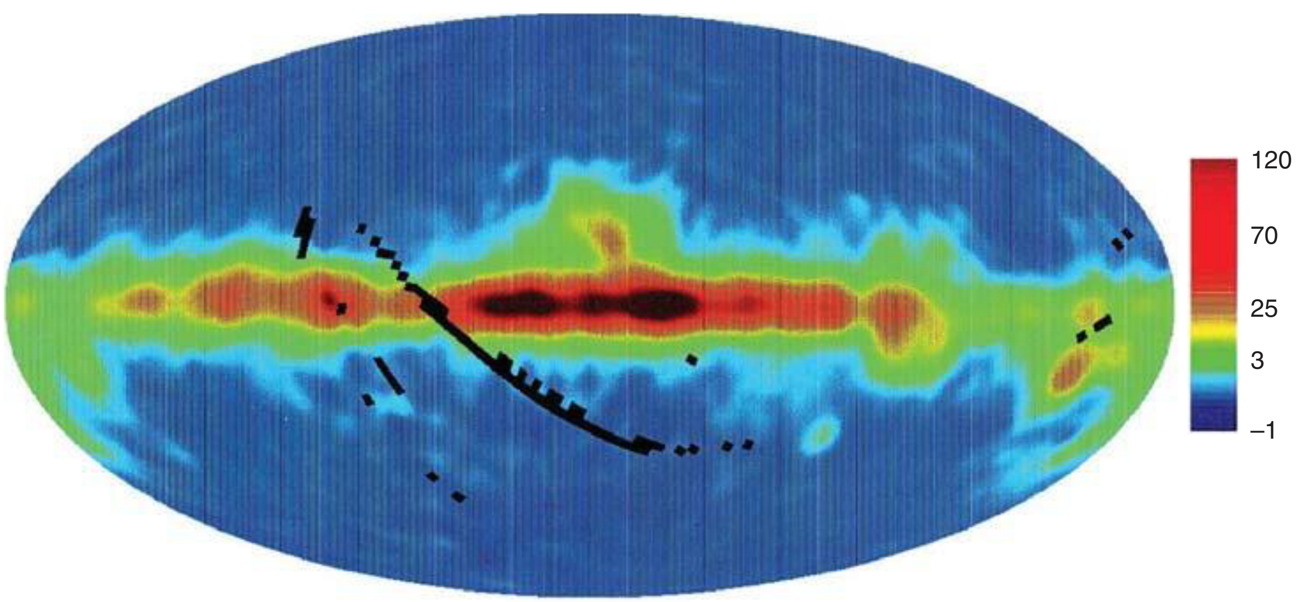
$$\frac{n_1}{n_0} = \frac{n_c k_{01}}{n_c k_{10} + A_{10}} \simeq n_c\frac{k_{01}}{A_{10}} = \frac{k_{01}}{k_{10}}\frac{n_c}{n_{\rm crit}} = 2e^{-91.21\,\mathrm{K}/T_{\rm gas}}\frac{n_{\rm H}}{n_{\rm crit}}$$

because $n_c \ll n_{\rm crit}$

$$\frac{n_1}{n_0} = 2e^{-91.21\,\mathrm{K}/T_{\rm exc}}$$

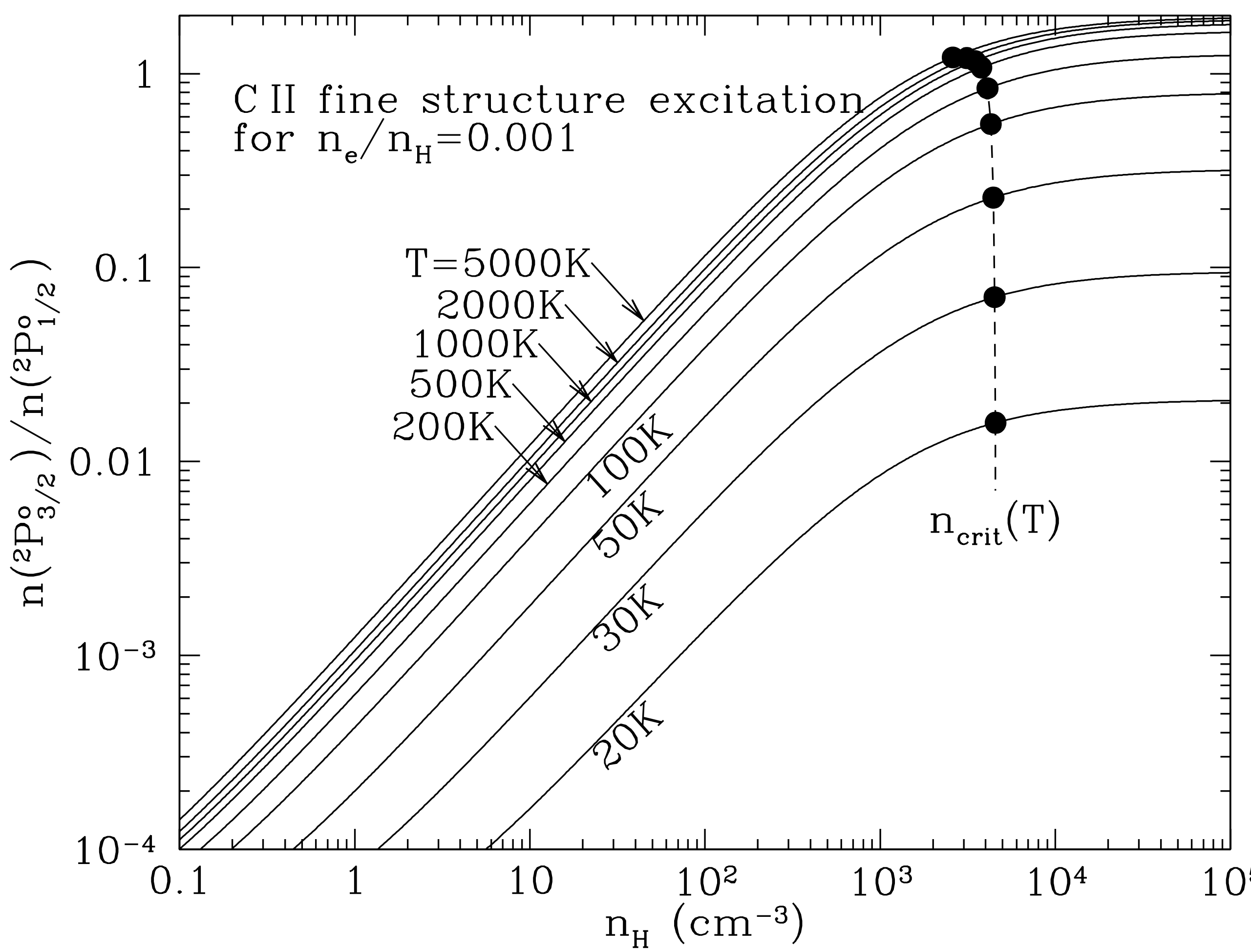
$$\rightarrow \frac{T_{\rm exc}}{T_{\rm gas}} \simeq \frac{1}{1 + (T_{\rm gas}/91.21\,\mathrm{K})\ln(n_{\rm crit}/n_{\rm H})} < 1$$

- The C II fine-structure levels will be sub-thermally excited. Collisional excitations of the upper level ${}^2P^o_{3/2}$ will usually be followed by radiative decays, removing energy from the gas.
- The [C II] 158 μm transition is the principal cooling transition for the diffuse gas in star-forming galaxies.




All-sky map of [C II] 158 μm emission, made by Far InfraRed Absolute Spectrophotometer (FIRAS) on the COsmic Background Explorer (COBE) satellite (Fixsen et al. 1999).

[Plate 3 in Draine]




[Fig. 17.4 in Draine]

# Equation for the 21-cm Spin Temperature

- We have derived the equation for the level populations in the presence of collision and radiation. Now, we will derive an intuitive equation for the spin temperature of the 21-cm line.

$$\frac{n_1}{n_0} = \frac{n_c k_{01} + \bar{n}_\gamma (g_1/g_0) A_{10}}{n_c k_{10} + (1+\bar{n}_\gamma) A_{10}}$$

- Let's define the temperature corresponding to the 21-cm transition.

$$T_* = E_{10}/k = 0.0682\,\mathrm{K}$$

- The temperatures of radiation and gas will be much higher than this:

$$T_{\rm gas} \approx 10 - 10^4\,\mathrm{K} \gg T_*, \quad T_{\rm rad} = 3.77\,\mathrm{K} \gg T_*, \quad T_{\rm spin} \gg T_*$$

- The population ratio can be written in terms of the excitation (spin) temperature:

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-T_*/T_{\rm spin}} \simeq \frac{g_1}{g_0}\left(1 - \frac{T_*}{T_{\rm spin}}\right)$$

- Similarly,

$$k_{01} = \frac{g_1}{g_0} k_{10} e^{-T_*/T_{\rm gas}} \simeq \frac{g_1}{g_0} k_{10}\left(1 - \frac{T_*}{T_{\rm gas}}\right)$$

$$\bar{n}_\gamma = \frac{1}{e^{T_*/T_{\rm rad}} - 1} \simeq \frac{T_{\rm rad}}{T_*}$$

- Substituting these into the population equation, we obtain

$$1 - \frac{T_*}{T_{\rm spin}} = \frac{n_c k_{10} \left(1 - T_*/T_{\rm gas}\right) + \left(T_{\rm rad}/T_*\right) A_{10}}{n_c k_{10} + \left(1 + T_{\rm rad}/T_*\right) A_{10}}$$

- Finally, we obtain the following equation:

$$T_{\rm spin} = \frac{T_* + T_{\rm rad} + y_c T_{\rm gas}}{1 + y_c} \qquad \Longleftarrow \qquad y_c \equiv \frac{T_*}{T_{\rm gas}} \frac{n_c k_{10}}{A_{10}}$$

- Ignoring $T_*$ term, we obtain an intuitive equation for the spin temperature.

$$\boxed{T_{\rm spin} = \frac{T_{\rm rad} + y_c T_{\rm gas}}{1 + y_c} \qquad \Longleftarrow \qquad y_c \equiv \frac{T_*}{T_{\rm gas}} \frac{n_c k_{10}}{A_{10}}}$$

This equation was first derived by G. Field (1958).

- This equation describes the spin temperature as ***a weighted mean of the radiation and gas temperatures with weights of 1 and $y_c$***.
- From the equation, we can show that

$$\boxed{\begin{array}{ll} T_{\rm spin} \simeq T_{\rm rad} & \text{if } \; y_c \ll 1 \\ T_{\rm spin} \simeq T_{\rm gas} & \text{if } \; y_c \gg 1 \end{array}}$$

- A new critical density of the colliding particle may be defined:

$$y_c = 1 \quad \Longrightarrow \quad n_{\rm crit}^* = \frac{T_{\rm gas}}{T_*}\frac{A_{10}}{k_{10}}$$

- Now, compare this density with the previous definition of the critical density.

$$\begin{aligned} n_{\rm crit} &\equiv \frac{[1+(n_\gamma)_{10}]\,A_{10}}{k_{10}} \\ &= \left[1+\frac{1}{e^{h\nu_{10}/kT_{\rm rad}}-1}\right]\frac{A_{10}}{k_{10}} \\ &\simeq \left(1+\frac{T_{\rm rad}}{T_*}\right)\frac{A_{10}}{k_{10}} \end{aligned}$$

$$\frac{n_{\rm crit}^*}{n_{\rm crit}} \approx \frac{T_{\rm gas}}{T_{\rm rad}}$$

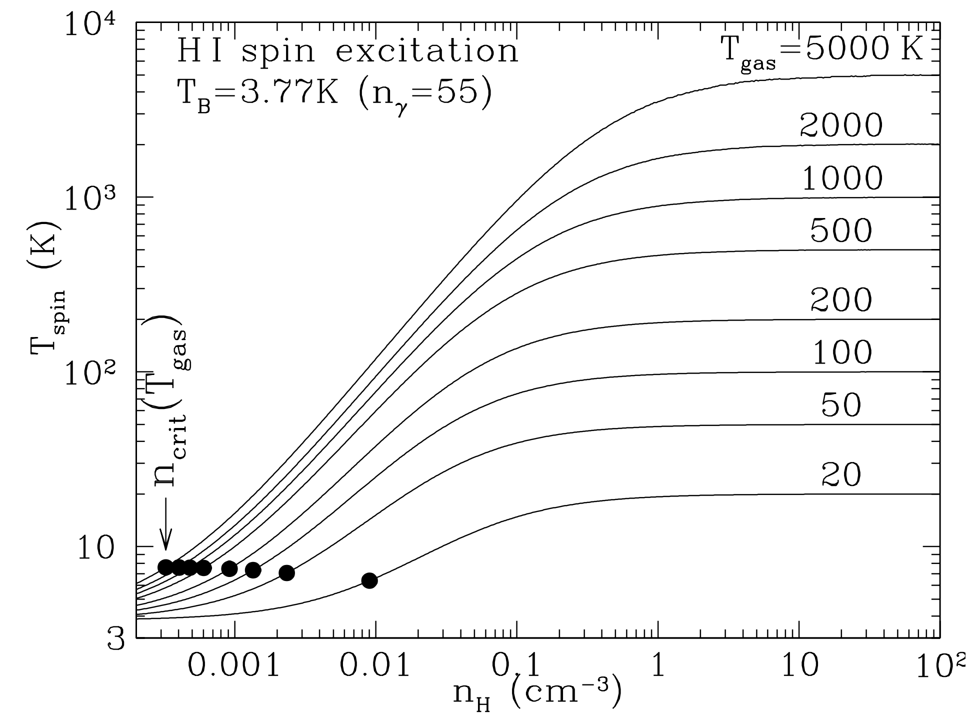
# Detectability of Hydrogen in a Low Density Medium

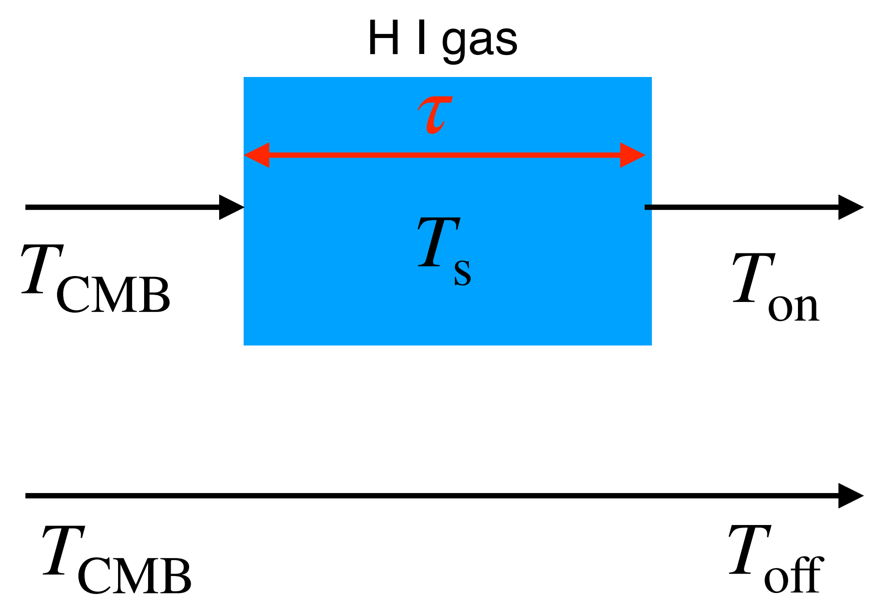
- In a very low density medium (WNM, CGM, IGM), the particle collisions are very rare ( $n_{\rm HI} \ll n_{\rm crit}$ ).
- The radiative transition due to the CMB photons will control the relative population between the hyperfine structures.
  - This indicates $T_{\rm s} = T_{\rm CMB}$.
  - The RT equation in the Rayleigh-Jeans regime can be written in terms of temperature:

$$T_{\rm on} = T_{\rm CMB} e^{-\tau} + T_{\rm s}\left(1 - e^{-\tau}\right) = T_{\rm CMB}$$
$$T_{\rm off} = T_{\rm CMB}$$
$$T_{\rm on} - T_{\rm off} = 0$$

  - Then, we have $T_{\rm on} = T_{\rm off} = T_{\rm CMB}$ .
  - ***Neither emission nor absorption feature from the hydrogen gas is detectable.***
- ***We need something that can make $T_{\rm s} \neq T_{\rm CMB}$.***

[Fig. 17.2 in Draine]



# The Wouthuysen-Field effect: The Third Mechanism controlling the Spin Temperature

- Wouthuysen (1952, AJ, 57, 31)

"Wouthuysen" is pronounced as roughly "Vowt-how-sen." (바우타이슨)

**Wouthuysen, S. A. On the excitation mechanism of the 21-cm (radio-frequency) interstellar hydrogen emission line.**

The mechanism proposed here is a radiative one: as a consequence of absorption and re-emission of Lyman-$\alpha$ resonance radiation, a redistribution over the two hyperfine-structure components of the ground level will take place. Under the assumption—here certainly permitted—that induced emissions can be negelcted, it can easily be shown that the relative distribution of the two levels in question, under stationary conditions, will depend solely on the shape of the radiation spectrum in the L$\alpha$ region, and not on the absolute intensity.

The shape of the spectrum of resonance radiation, quasi-imprisoned in a large gas cloud, could only be determined by a careful study of the "scattering" process (absorption and re-emission) in a cloud of definite shape and dimensions. The spectrum will turn out to depend upon the localization in the cloud.

Some features can be inferred from more general considerations. Take a gas in a large container, with perfectly reflecting walls. Let the gas be in equilibrium at temperature $T$, together with Planck radiation of that same temperature. The scattering processes will not affect the radiation spectrum. One can infer from this fact that the photons, after an infinite number of scattering processes on gas atoms with kinetic temperature $T$, will obtain a statistical distribution over the spectrum proportional to the Planck-radiation spectrum of temperature $T$. After a finite but large number of scattering processes the Planck shape will be produced in a region around the initial frequency.

Photons reaching a point far inside an interstellar gas cloud, with a frequency near the L$\alpha$ resonance frequency, will have suffered on the average a tremendous number of collisions. Hence in that region, which is wider the larger the optical depth of the cloud is for the Lyman radiation, the Planck spectrum corresponding to the gas-kinetic temperature will be established as far as the shape is concerned. Because, however, the relative occupation of the two hyperfine-structure components of the ground state depends only upon the shape of the spectrum near the L$\alpha$ frequency, this occupation will be the one corresponding to equilibrium at the gas temperature.

The conclusion is that the resonance radiation provides a long-range interaction between gas atoms, which forces the internal (spin-)degree of freedom into thermal equilibrium with the thermal motion of the atoms.

*Institute for Theoretical Physics of the City University, Amsterdam.*

**From a thermodynamic argument, Wouthuysen speculated the followings:**

***A tremendous number of scattering will establish the Planck-like spectrum, at the Ly$\alpha$ line center, corresponding to the gas-kinetic temperature.***

The Ly$\alpha$ radiation is coupled with the hyperfine state of the hydrogen atom.

In the end, ***the 21cm spin temperature will become equal to the kinetic temperature of the hydrogen gas.***

# Mechanisms that controls the spin temperature

- The spin temperature ($T_s$) is determined by three mechanisms.

(1) Direct Radiative Transitions by the background radiation field (Cosmic Microwave Background or Galactic Synchrotron)

$$I_\nu = \frac{2k_B T_R}{\lambda^2}$$

(Rayleigh-Jeans Law)

$$T_R = \text{brightness temperature}$$
$$= 2.73\,\text{K or } 3.77\,\text{K}$$

(2) Collisional Transitions (collision with other hydrogen and electron)

$$T_K = \text{gas kinetic temperature}$$

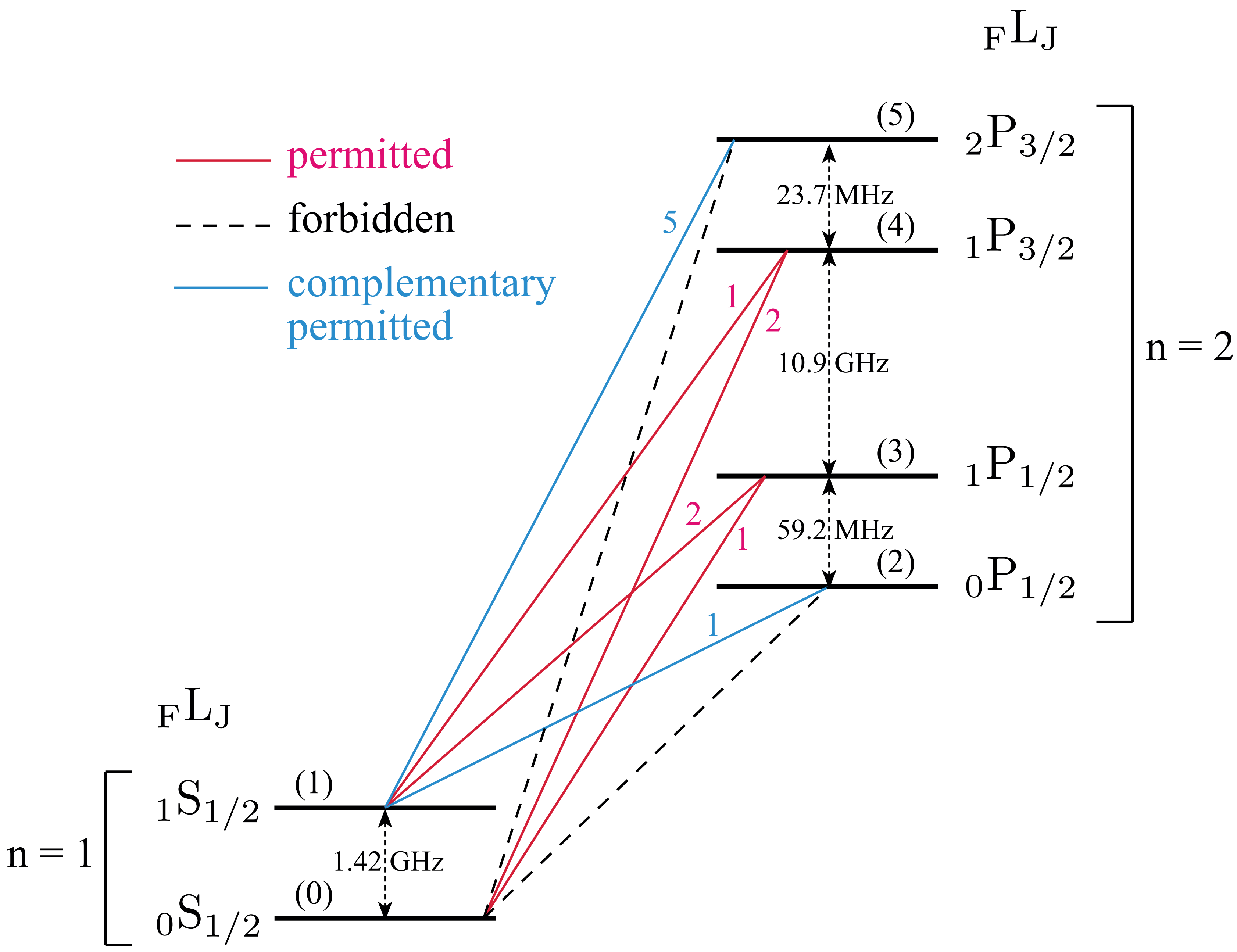
(3) Lyα pumping: Indirect Radiative Transitions involving intermediate levels caused by Lyα resonance scattering

$$T_\alpha = \text{color temperature}$$

$$J(\nu) \propto \exp\left(-\frac{h\nu}{k_B T_\alpha}\right)$$

# Indirect Level Population by Ly$\alpha$ Scattering

***The WF effect is a mechanism that the resonance scattering of Ly$\alpha$ photons indirectly control the relative populations between the hyperfine levels in the ground state (n = 1) via transitions involving the n = 2 state as an intermediate state.***



# Equation for spin temperature

We obtain the following equation for the spin temperature in terms of the 21 cm brightness temperature, gas kinetic temperature, and Ly-alpha color temperature:

$$T_{\rm S} = \frac{T_* + T_R + y_c T_K + y_\alpha T_\alpha}{1 + y_c + y_\alpha}$$

$$T_{\rm S} \simeq \frac{T_R + y_c T_K + y_\alpha T_\alpha}{1 + y_c + y_\alpha}$$

where $$y_c \equiv \frac{T_*}{T_K}\frac{P_{10}^c}{A_{10}}$$

$$y_\alpha \equiv \frac{T_*}{T_\alpha}\frac{P_{10}^\alpha}{A_{10}}$$

$$T_* = \frac{h\nu_{10}}{k_B} = 0.0681\ ^\circ\mathrm{K}$$ (This term is negligible in the above equation.)

***Two requirements for the WF effect:***

(1) $J_\nu \propto \exp\left(-\frac{h\nu}{kT_\alpha}\right)$ with $T_\alpha = T_{\rm K}$

(2) $y_\alpha \gg 1$ and $y_\alpha \gg y_c$

(Ly$\alpha$ radiation field should be strong.)

Read Seon & Kim (2020) for a details.
https://ui.adsabs.harvard.edu/abs/2020ApJS..250....9S/abstract

# H II Regions

- Ionization and Recombination
- Strömgren Sphere
- Recombination Lines
- Heating & Cooling

# Atomic Processes

- **Excitation and de-excitation (Transition)**
  - Radiative excitation (photoexcitation; photoabsorption)
  - Radiative de-excitation (spontaneous emission and stimulated emission)
  - Collisional excitation
  - Collisional de-excitation

- **Emission Line**
  - Collisionally-excited emission lines
  - Recombination lines (recombination following photoionization or collisional ionization)

- **Ionization**
  - Photoionization and Auger-ionization
  - Collisional Ionization (Direct ionization and Excitation-autoionization)
- **Recombination**
  - Radiative recombination $\Longleftrightarrow$ Photoionization
  - Dielectronic Recombination (not dielectric!)
  - Three-body recombination $\Longleftrightarrow$ Direct collisional ionization
- **Charge exchange**

# Ionization - [Photoionization]

- Interstellar medium (ISM) is transparent to $h\nu < 13.6$ eV photons, but is very opaque to ionizing photons with $h\nu > 13.6$ eV. In fact, the ISM does not become transparent until $h\nu \sim 1$ keV.
  - Sources of ionizing photons include massive, hot young stars, hot white dwarfs, and supernova remnant shocks.
- ***From the Outer Shells***
  - Photoionization is the ionization of an atomic species by the absorption of a photon. Photoionization is the inverse process to radiative recombination.

$$X + h\nu \;\rightarrow\; X^+ + e^- + \Delta E$$

  - If the incoming photon has sufficient energy, it may leave the ionized species in an excited state.

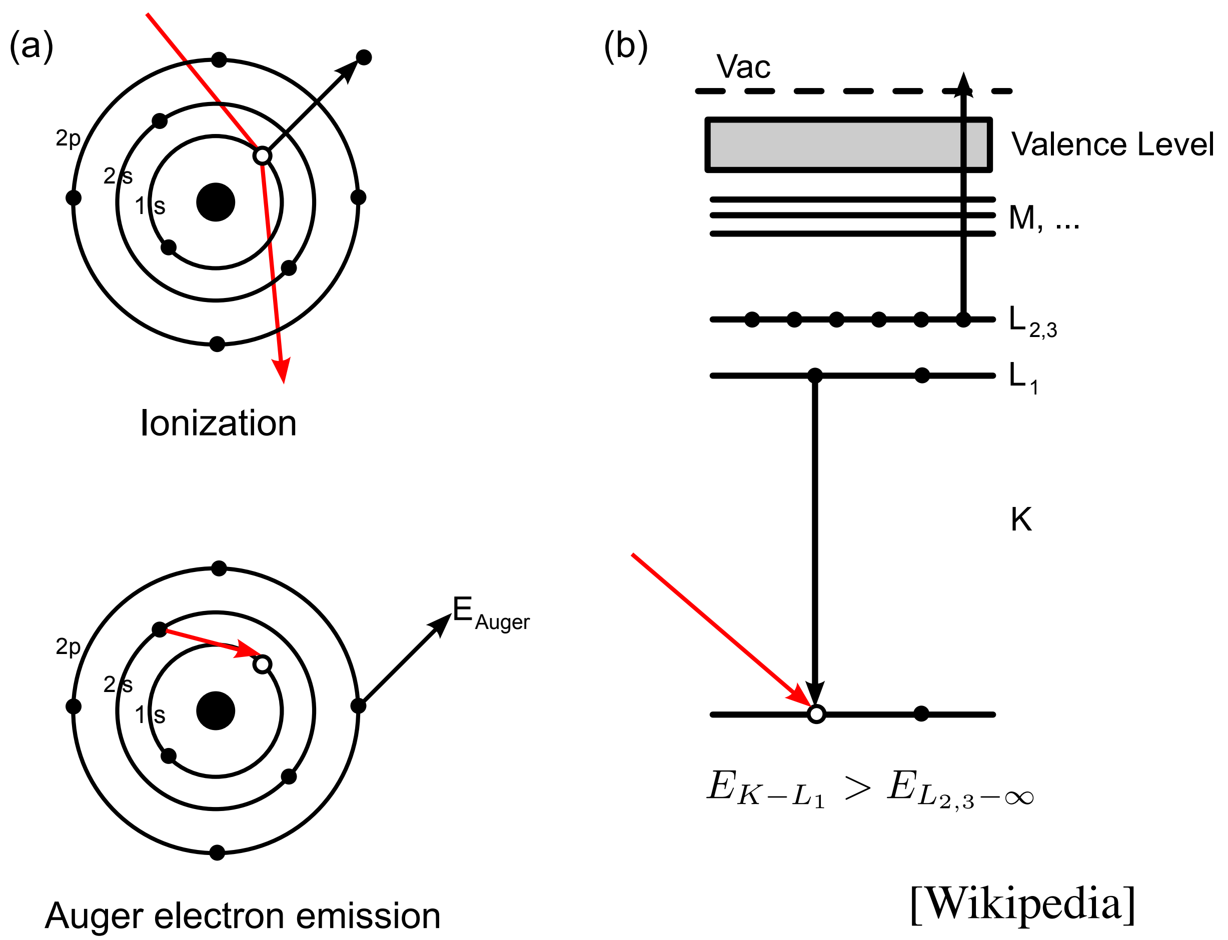
$$\begin{aligned} X + h\nu \;&\rightarrow\; X_*^+ + e^- + \Delta E \\ X_*^+ \;&\rightarrow\; X^+ + h\nu_1 + h\nu_2 + \cdots \end{aligned}$$

Here, $X$ denotes an atom, molecule, or ion and subscript * indicates an excited states.

$\Delta E$ denotes energy carried by the electron.

- ***Inner Shell Photoionization***
    - If the energy of the incoming photon is still higher, it becomes possible to remove one of the inner shell electrons which also results in a change in the electron configuration in the excited species. This may be followed by a radiative readjustment back to the ground state.
    - However, in this case, **Auger ionization** becomes more probable. High energy photons may eject an inner shell electron from an ion or atom, and the resulting ion may then fill the gap in its inner shell with an outer electron, ejecting another outer electron to remove the energy in a ***radiationless*** transition called an Auger transition. Such a process will produce very energetic electrons which will lose their energy in heating up the gas.

$$
\begin{aligned}
X + h\nu &\rightarrow X_{**}^{+} + e^{-} + \Delta E_1 \\
X_{**}^{+} &\rightarrow X_{*}^{(m+1)+} + me^{-} + \Delta E_2 \\
&\quad \text{(radiationless autoionization, } m \geq 1\text{)} \\
X_{*}^{(m+1)+} &\rightarrow X^{(m+1)+} + h\nu_1 + h\nu_2 + \cdots
\end{aligned}
$$




[Wikipedia]

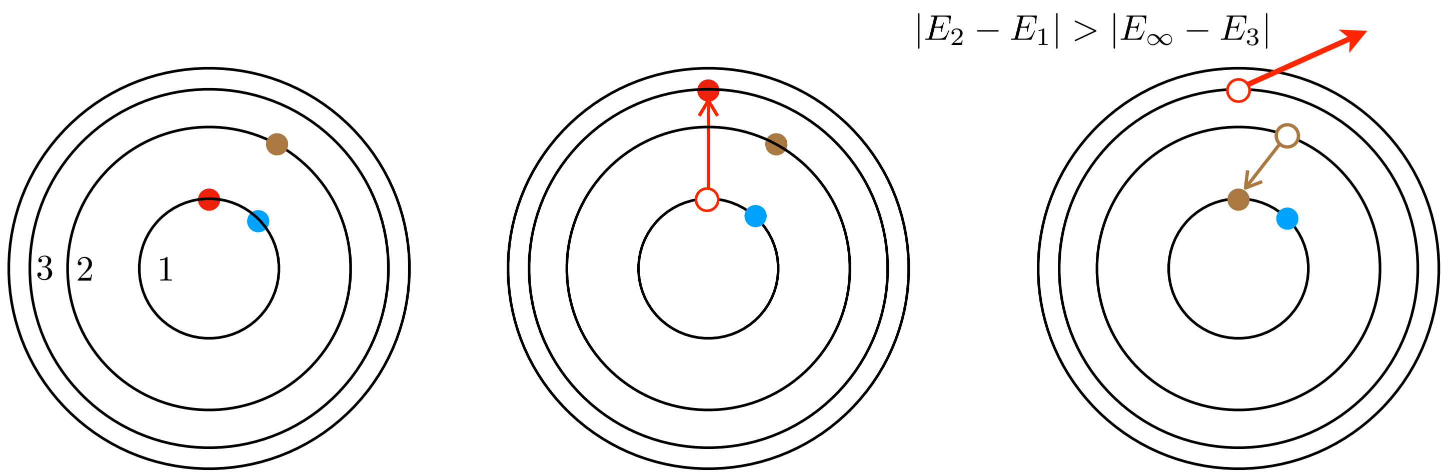
# Ionization - [Collisional ionization]

- ***Direct collisional ionization***: The process whereby an electron strikes an atom or ion X, with sufficient energy to strip out a bound electron:

$$X + e^- \;\rightarrow\; X^+ + 2e^- - \Delta E$$

- ***Excitation-autoionization***: At sufficiently high electron impact energies, more than one electron of the target may be excited, leaving the atom in an unstable state, which is stabilized by the radiationless ejection of one of outer electrons, possibly followed by a radiative decay of the ionized atom back go its ground state. This process is favored in heavy elements which have a large number of inner shell electrons and only one or two electrons in the outer shell.

$$\begin{aligned} X + e^- &\;\rightarrow\; X_* + e^- - \Delta E_1 \\ X_* &\;\rightarrow\; X_*^+ + e^- + \Delta E_2 \\ X_*^+ &\;\rightarrow\; X^+ + h\nu \end{aligned}$$




- For example, in collisions of Li-like ions, excitation and autoionization can occur via excitation of the 1s-electron into states with principal quantum numbers $n \geq 2$. After the decay of a doubly excited state, one has an additional electron in the final channel.

$$\begin{aligned} X(1s^2 2s) + e^- &\;\rightarrow\; X_{**}(1s2snl) + e^- - \Delta E_1 \\ &\;\rightarrow\; X_*^+(1s^2) + (e^- + \Delta E_2) + (e^- - \Delta E_1) \end{aligned}$$

# Recombination

- ***Radiative recombination***
  - Radiative recombination is the process of capture of an electron by an ion where the excess energy of the electron is radiated away in a photon.
  - The electron is captured into an excited state. The recombined but still excited ion radiates several photons in a radiative cascade, as it returns to the ground state:

$$\begin{aligned} X^+ + e^- &\rightarrow X_* + h\nu && \text{(recombination continuum)} \\ X_* &\rightarrow X + h\nu_1 + h\nu_2 + h\nu_3 + \cdots && \text{(recombination lines)} \end{aligned}$$

  - The photon in the first line represents a **recombination continuum** ($h\nu$) photon. However, photons ($h\nu_1$, $h\nu_2$, $h\nu_3$) represent quantized transitions and are therefore termed **recombination lines**.
  - ***The total effective recombination rate is the sum of the recombination rate to each state.***

- ***Dielectronic recombination***
  - For an electron that is initially free to be captured to a bound state of an atom or ion, the electron must lose energy.
    - Radiative recombination is relatively slow because it is necessary to create a photon to remove this energy as part of the capture process. This can take place only during the brief time that the free electron is appreciably accelerated by the electric field of ion.
    - However, if an ion has at least one bound electron, then it is possible for the incoming electron to transfer energy to a bound electron, promoting the bound electron to an excited state, and removing enough energy from the first electron that it too can be captured in an excited state. Then, the ion now have two electrons in excited state.
    - Dielectronic recombination (DR) is a resonant two-step process.
      - The first step is a double-electron process, often called dielectronic capture, through which one free electron is captured and another core electron is simultaneously excited forming a doubly excited state. One of the electron is in an autoionizing state, $n_1l_1$, and the other is in an excited state, $n_2l_2$. In the second step, the ion in a doubly excited state emits a photon and decays into a stable state below the ionization limit.

$$
\begin{aligned}
X^+(1s,\ldots) + e^- &\rightarrow X_{**}(n_1l_1;\ n_2l_2) \\
X_{**}(n_1l_1;\ n_2l_2) &\rightarrow X_*(n_3l_3;\ n_2l_2) + h\nu \\
X_*(n_3l_3;\ n_2l_2) &\rightarrow X(n_3l_3;\ n_4l_4) + h\nu_1 + h\nu_2 + \cdots
\end{aligned}
$$

    - Dielectronic recombination is important in high-temperature plasmas, where it often exceeds the radiative recombination rate.

- Three-body Recombination
  - The combination process of an electron with a positive ion in a gas in such a way that the incoming free electron transfers energy and momentum to another free electron in the neighborhood of the ion.

$$X^+ + 2e^- \;\rightarrow\; X + e^- + \Delta E$$

  - Three-body recombination is the inverse process of collisional ionization.
  - In most interstellar medium, three-body recombination is unimportant.
  - In dense regions with electron densities above $10^4$ cm$^{-3}$, three-body recombination into high levels of the hydrogen atom with principal quantum numbers ($n > 100$) can be important. (Compare Eq. (3.44) and (14.6) in Draine)

# Charge Exchange

- During the collision of two ionic species, the charge clouds surrounding each interact, and it is possible that an electron is exchanged between them.

$$X^+ + Y \rightleftharpoons X + Y^+ + \Delta E$$

  - Since, in virtually all diffuse astrophysical plasmas, hydrogen and helium are overwhelmingly the most abundant species, the charge-exchange reactions which are significant to the ionization balance of the plasma are

$$\begin{aligned} X^+ + \mathrm{H}^0 &\rightleftharpoons X + \mathrm{H}^+ + \Delta E \\ X^+ + \mathrm{He}^0 &\rightleftharpoons X + \mathrm{He}^+ + \Delta E \end{aligned}$$

  - The reactions are exothermic (발열) because of the lower ionization potential of the $X^+$ ion. Thus, the reverse reaction occurs only when $kT \gtrsim \Delta E$. In many cases we have to consider only the forward reaction.
  - Charge-exchange may also occur in collisions of molecules with atoms. i.e.,

$$\mathrm{CO}^+ + \mathrm{H}^0 \rightarrow \mathrm{CO} + \mathrm{H}^+$$

# Ionization Equilibrium

- ***Photoionization Equilibrium***:
  - Balance between photo-ionization and the process of recombination.

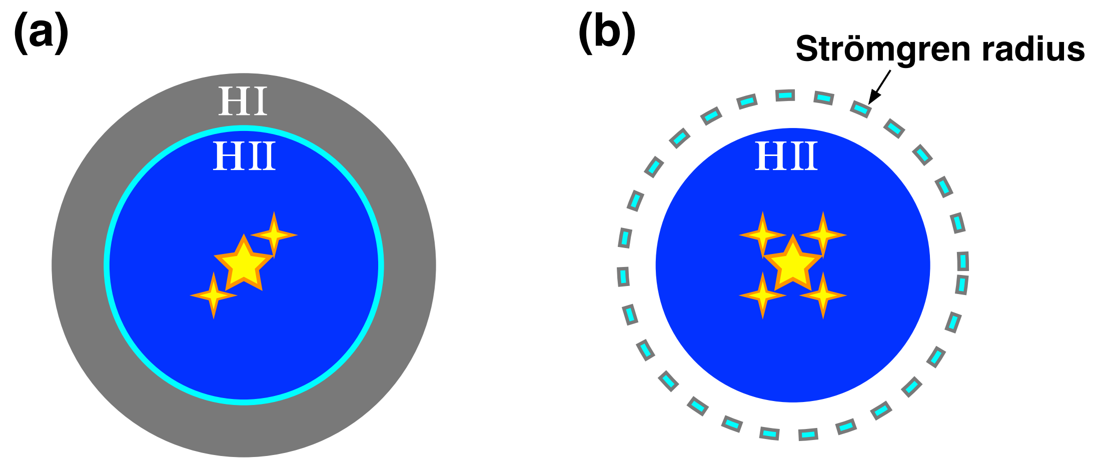
- ***Collisional Ionization Equilibrium (CIE)*** or coronal equilibrium
  - Balance at a given temperature between collisional ionization from the ground states of the various atoms and ions, and the process of recombination from the higher ionization stages.
  - In this equilibrium, effectively, all ions are in their ground state.

- Ionization balance under conditions of local thermodynamic equilibrium (LTE)
  - The ionization equilibrium in LTE is described by the ***Saha equation***.
  - In LTE, the excited states are all populated according to Boltzmann's law.

# Ionization bounded vs. Density bounded

- Ionized atomic hydrogen regions, broadly termed "**H II regions**", are composed of gas ionized by photons with energies above the hydrogen ionization energy of 13.6 eV.
  - ***Ionization Bounded:*** These objects include "***classical H II regions***" ionized by hot O or B stars (or clusters of such stars) and associated with regions of recent massive-star formation, and "planetary nebulae", the ejected outer envelopes of AGB stars photoionized by the hot remnant stellar core.
  - ***Density Bounded (Matter Bounded): Warm Ionized Medium / Diffuse Ionized Gas***: Ionized Gas in the diffuse ISM, far away from OB associations.

(a) An ionization-bounded nebula whose radius is determined by the ionization equilibrium. The LyC is entirely consumed to ionized the surrounding H I gas.
(b) In a density-bounded nebula, the amount of the surrounding H I gas is not enough to consume all LyC photons. Some of the LyC escapes from the cloud, which is called the LyC leakage.



- The UV, visible and IR spectra of H II regions are very rich in emission lines, primarily collisional excited lines of metal ions and recombination lines of hydrogen and helium. H II regions are also observed at radio wavelengths, emitting radio free-free emission from thermalized electrons and radio recombination lines from highly excited states of H, He, and some metals (e.g., H109$\alpha$ and C lines).

- Three processes govern the physics of H II regions:
  - ***Photoionization Equilibrium***: the balance between photoionization and recombination. This determines the spatial distribution of ionic states of the elements in the ionized zone.
  - ***Thermal Balance*** between heating and cooling. Heating is dominated by photoelectrons ejected from hydrogen and helium with thermal energies of a few eV. Cooling is mostly dominated by electron-ion impact excitation of metal ion followed by emission of “forbidden” lines from low-lying fine structure levels. It is these cooling lines that give H II regions their characteristic spectra.
  - ***Hydrodynamics***, including shocks, ionization and photodissociation fronts, and outflows and winds from the embedded stars.

- Ionization fraction
  - In the CNM, the fractional ionization $x = n_e/n_{\rm H} \sim 0.001$ . In the WNM, $x \sim 0.1$ .
  - Although the number of free electrons are small in the neutral ISM, free electrons play a role in bringing the WNM to kinetic equilibrium. Free electrons photo-ejected from dust grains are the major heat source in the neutral medium.
  - In the WIM, $x \sim 0.7$ . In the HIM, $x \sim 1.0$ .

- H II regions, with $T \sim$ 10,000 K and $n \sim$ 0.3 cm$^{-3}$, contributes only a few percent of the mass of the ISM, and no more than ten percent of its volume.
  - Classical H II regions and planetary nebulae have a similar temperature, but planetary nebulae have a higher density.

# Ionization Energy

- First ionization energy = energy required to remove the most loosely bound electron in a neutral atom in its ground state.
  - The first ionization energy of hydrogen is $I_{\rm H} = 13.59844\,{\rm eV}$.
  - The highest first ionization energy is that of He, with $I_{\rm He} = 24.6\,{\rm eV} = 1.91 I_{\rm H}$.
  - The second ionization energy of helium is $I_{\rm HeII} = 54.4\,{\rm eV} = 4 I_{\rm H}$.
  - The lowest first ionization energy of astrophysically interesting elements is that of potassium (K, $Z$ = 19), with $I_{\rm K} = 4.3\,{\rm eV}$. (Rubidium, cesium, and francium have lower first ionization energies, but they are not much seen in the ISM.)
  - Thus, photoionization of neutral atoms will be done by UV photons in the wavelength range $\lambda$ = 500 - 3000Å (corresponding to $E$ = 4.1 - 24.8 eV).

- A hydrogenic (hydrogen-like) ion with atomic number Z has an ionization energy of $Z^2 I_{\rm H}$.
  - 54.4 eV for He II, 122.4 eV for Li III, and so forth.

# Photoionization

- The (nonrelativistic) quantum mechanics of hydrogen-like ions (with only one electron) give an analytic expression for the ground-state photoionization (photoelectric) cross section.

$$\boxed{\sigma_{\rm pi}(\nu) = \sigma_0 \left(\frac{Z^2 I_{\rm H}}{h\nu}\right)^4 \frac{{\rm e}^{4-4\arctan(x)/x}}{1-{\rm e}^{-2\pi/x}}, \quad x \equiv \sqrt{\frac{h\nu}{Z^2 I_{\rm H}} - 1} \quad \text{for } h\nu > Z^2 I_{\rm H}}$$

  - The cross section at threshold is

$$\sigma_0 \equiv \frac{2^9\pi}{3{\rm e}^4}\alpha\pi a_0^2 Z^{-2} = 6.304\times10^{-18} Z^{-2}\,{\rm cm}^{-2}$$

fine-structure constant
$(\alpha \equiv e^2/hc = 1/137.04, \ {\rm e} = 2.71828...)$

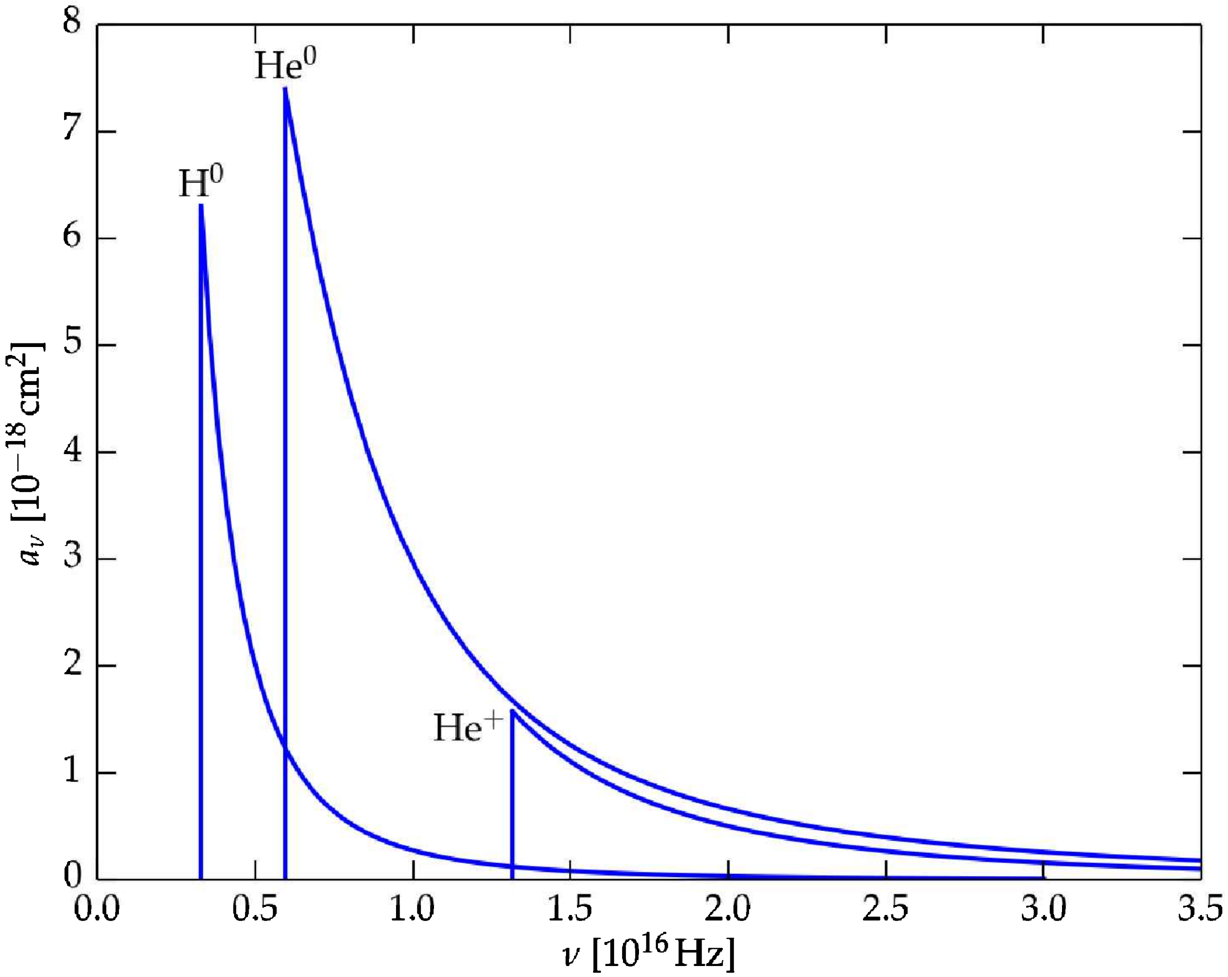
  - The photoionization cross section is reasonably approximated by a power-law:

$$\boxed{\sigma_{\rm pi}(\nu) \approx \sigma_0 \left(\frac{h\nu}{Z^2 I_{\rm H}}\right)^{-3} \quad \text{for } Z^2 I_{\rm H} \lesssim h\nu \lesssim 100 Z^2 I_{\rm H}}$$

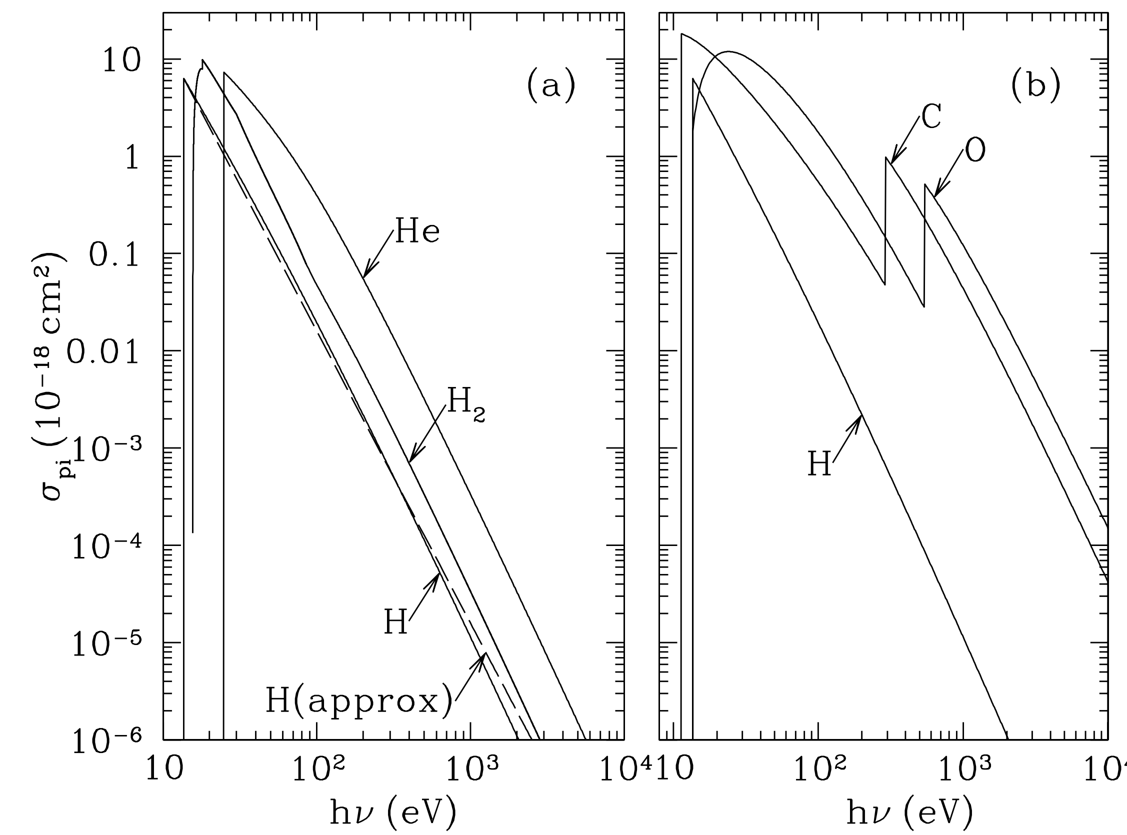
  - At high energies, the asymptotic behavior is:

$$\sigma_{\rm pi}(\nu) \approx \frac{2^8}{3Z^2}\alpha(\pi a_0^2)\left(\frac{h\nu}{Z^2 I_{\rm H}}\right)^{-3.5} \quad \text{for } \ h\nu \gg Z^2 I_{\rm H}$$

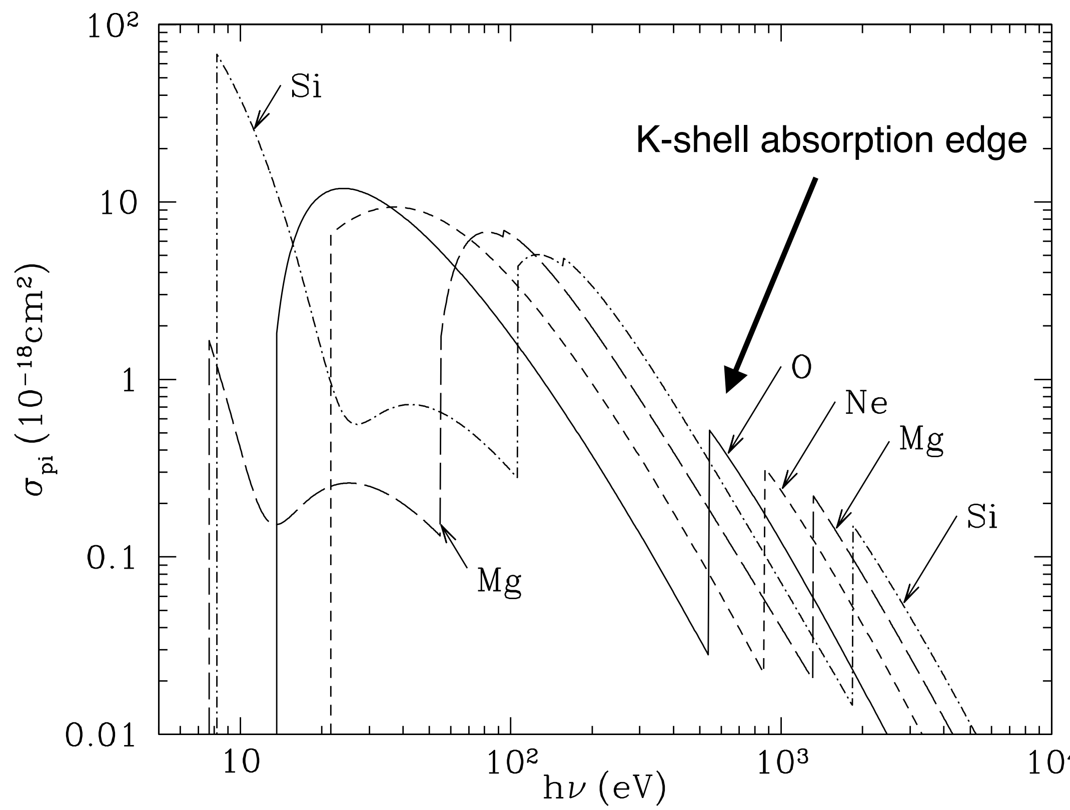
  The hydrogen photoionization cross section becomes equal to the Thomson (Compton) Scattering cross section for $h\nu \approx 2.5\,{\rm keV}$ ; above this energy photoionization of H is dominated by Thomson scattering rather than photoelectric absorption.




Photoionization cross section for hydrogen(H$^0$), hydrogenic helium (He$^+$), and neutral helium (He$^0$).
[Fig. 4.1 in Ryden]




[Fig. 13.1 in Draine]




[Fig. 13.2 in Draine]

- For atoms with three or more electrons, the energy dependence of the photoionization cross section is considerably more complicated because there is more than one available channel.
  - Convenient analytic fits to the contribution of individual shells to photoionization cross section are given by Verner & Yakovlev (1995) and Verner et al. (1996).

- Photoionization rate (the probability of photoionization per unit time, for a single atom that undergoes photoionization)

$$\zeta_{\rm pi} = \int_{\nu_1}^{\infty} \sigma_{\rm pi}(\nu) c \frac{u_\nu}{h\nu} d\nu \qquad \nu_1 = Z^2 I_{\rm H}/h = 3.29 \times 10^{15}\,{\rm Hz}\ (\text{for hydrogenic ions})$$

= (cross section) x (flux of ionizing photons by number)

$$\text{flux} = 4\pi \frac{J_\nu}{h\nu} = c\frac{u_\nu}{h\nu}$$

- Since the photoionization cross section decreases fairly steeply with increasing photon energy, **most photoionization occurs by photons with energies just above the ionization energy (13.6 eV for hydrogen)**, in a range of the spectrum where the background is produced mainly by hot stars.
- The volumetric photoionization rate, for instance, for hydrogen is

$$\frac{dn_p}{dt} = n_{\rm H^0}\zeta_{\rm pi} \quad [{\rm cm^{-3}\,s^{-1}}]$$

= (# of atoms/volume) x (ionization rate per atom)

where $n_p$ and $n_{\rm H^0}$ are the number density of proton (ionized hydrogen) and the number of neutral hydrogen atom, respectively.

# Radiative Recombination (RR)

- The cross section for the radiative recombination can be obtained using the photoionization cross section and the ***Milne relation***, which is derived from the principle of detailed balance.
- Consider an ion with its electron in some configuration that we will refer to as the "core". In a low-density plasma, free electrons can undergo transitions to bound states by emission of a photon. The electron is captured into some specific state $n\ell$ that will initially unoccupied.

$$X^{+}(\text{core}) + e^{-} \;\rightarrow\; X(\text{core} + n\ell) + h\nu$$

- The RR rate coefficient for electron capture directly to level $n\ell$, with emission of a photon of energy $h\nu = I_{n\ell} + E$ (where $I_{n\ell}$ is the bounding energy required for ionization from level $n\ell$ and $E$ is the captured electron energy), is

$$\alpha_{n\ell}(T) \equiv \langle \sigma_{\mathrm{rr,n\ell}} v \rangle = \left(\frac{8kT}{\pi m_e}\right)^{1/2} \int_0^\infty \sigma_{\mathrm{rr},n\ell}(E) \frac{E}{kT} e^{-E/kT} \frac{dE}{kT}$$

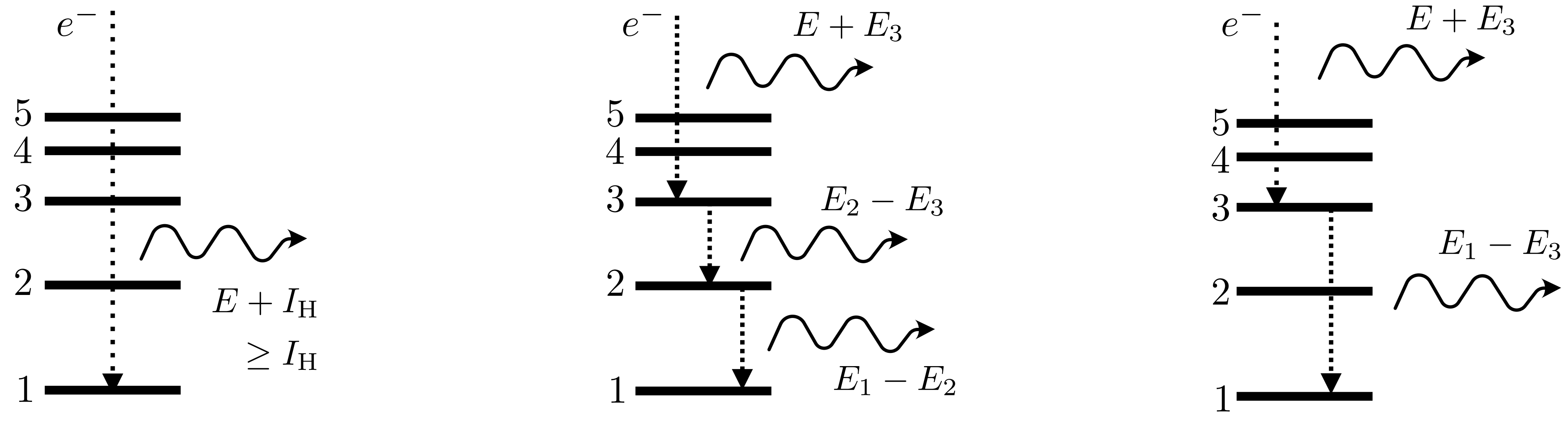
The integral indicates an average over the Maxwell distribution for electrons.

- The volumetric rate of RR, for instance for hydrogen, can be written as

$$\frac{dn_p}{dt} = -n_e n_p \alpha$$

Notice that an electron of any energy can trigger a collisional de-excitation as well as RR.

- Properties of radiative recombination
  - In general, *$\alpha_{n\ell}$ is a decreasing function of $T$*, although it depends weakly on temperature. Therefore, ***it's easier to recombine with a slow electron than with a fast electron.***
  - In general, $\alpha_n = \sum_\ell \alpha_{n\ell}$ summed over all applicable values of $\ell$, is a decreasing function of $n$, implying that ***it's easier to recombine to a low energy level than to a high energy level.***
- ***Recombination to the ground state:***
  - If the recombination is to the ground state of hydrogen ($n = 1$), the energy of the emitted photon is $E + I_{\rm H} \geq I_{\rm H}$ . Thus, the emitted photon is guaranteed to have an energy of at least 13.6 eV, and will be capable of photoionizing any neutral hydrogen atom that it encounters. Thus, in regions that are optically thick to UV light at photon energies just above $I_{\rm H}$, the emitted photon will be rapidly destroyed in photoionizing a nearby hydrogen atom.



# Case A and B (Radiative Recombination of Hydrogen)

- ***On-the-spot approximation:***
  - In optically thick regions, it is assumed that every photon produced by radiative recombination to the ground state of hydrogen is immediately, then and there, destroyed in photoionizing other hydrogen atom.
  - In the on-the-spot approximation, recombination to the ground state has no net effect on the ionization state of the hydrogen gas.
- Baker & Menzel (1938) proposed two limiting cases:
  - ***Case A:*** ***Optically thin*** to ionizing radiation, so that every ionizing photon emitted during the recombination process escapes. For this case, we sum the radiative capture rate coefficient $\alpha_{n\ell}$ over all levels $n\ell$.
  - ***Case B:*** ***Optically thick*** to radiation just above $I_{\rm H} = 13.60\,{\rm eV}$, so that ionizing photons emitted during recombination are immediately reabsorbed, creating another ion and free electron by photoionization. In this case, the recombinations directly to $n = 1$ do not reduce the ionization of the gas: ***only recombinations to*** $n \geq 2$ ***act to reduce the ionization.***

  - **Case B in photoionized gas**: Photoionized nebulae around OB stars (H II regions) usually have large enough densities of neutral H. For this situation, case B is an excellent approximation.
  - **Case A in collisionally ionized gas**: Regions where the hydrogen is collisional ionized are typically very hot ($T > 10^6$ K) and contain a very small density of neutral hydrogen. For these shock-heated regions, case A is an excellent approximation.

- ***Radiative recombination rate coefficients***:
    - In Case A, the relevant radiative recombination rate coefficient is found by summing over all energy levels of the hydrogen atom:

$$\alpha_{\mathrm{A,H}}(T) \equiv \sum_{n=1}^{\infty} \sum_{\ell=0}^{n-1} \alpha_{n\ell}(T)$$

$$\approx 4.18 \times 10^{-13}\, T_4^{-0.721-0.021 \ln T_4} \;[\mathrm{cm^3\, s^{-1}}] \;\; \text{for } 0.3 \lesssim T_4 \lesssim 3 \quad \left(T_4 \equiv T/10^4\,\mathrm{K}\right)$$

    - In Case B, the relevant radiative recombination rate coefficient is found by summing over all energy levels other than the ground state:

$$\alpha_{\mathrm{B,H}}(T) \equiv \sum_{n=2}^{\infty} \sum_{\ell=0}^{n-1} \alpha_{n\ell}(T) = \alpha_{\mathrm{A,H}}(T) - \alpha_{1\mathrm{s}}(T)$$

$$\approx 2.59 \times 10^{-13}\, T_4^{-0.833-0.034 \ln T_4} \;[\mathrm{cm^3\, s^{-1}}] \;\; \text{for } 0.3 \lesssim T_4 \lesssim 3$$

    - The percentage of radiative recombinations that go directly to the ground state is 30% at $T$ = 3000 K but increases to 46% at $T$ = 30,000 K. Thus, the distinction between Case A and Case B becomes increasingly important at higher temperatures.

$$\frac{\alpha_{1\mathrm{s,H}}}{\alpha_{\mathrm{A,H}}} = 1 - \frac{\alpha_{\mathrm{B,H}}}{\alpha_{\mathrm{A,H}}} = 1 - 0.0619\, T_4^{-0.112-0.013 \ln T_4}$$

# H II Regions and Strömgren Spheres

- ***Strömgren Sphere***:
  - Following Strömgren (1939), we consider the simple idealized problem of a fully ionized, spherical region of uniform medium plus a central source of ionizing photons.
  - The ionization is assumed to be maintained by absorption of the ionizing photons radiated by a central hot star. The central source produces ionizing photons, with energy $\nu > \nu_0 = I_{\rm H}/h$ at a constant rate $Q_0$ [photons s$^{-1}$].
  - At a distance $r$ from the central star, the balance equation between ionization and recombination balance is

$$n_{\rm H^0}\int_{\nu_0}^{\infty}\frac{4\pi J_\nu}{h\nu}\sigma_\nu d\nu = n_p n_e \alpha_{\rm B,H}$$

$L_\nu$ = luminosity of the central star at frequency $\nu$.

From the RT equation,

$$4\pi J_\nu = \underbrace{\frac{L_\nu}{4\pi r^2}e^{-\tau_\nu}}_{\text{geometrical attenuation + radiative absorption}}, \quad \text{where } \tau_\nu = \int_0^r n_{\rm H^0}\sigma_\nu dr$$

Integrating the balance equation over the whole volume:

$$\int_0^\infty\int_{\nu_0}^\infty \frac{L_\nu/h\nu}{4\pi r^2}e^{-\tau_\nu} n_{\rm H^0}\sigma_\nu d\nu(4\pi r^2)dr = \int_0^\infty n_p n_e \alpha_{\rm B,H}(4\pi r^2)dr$$

$$\int_{\nu_0}^\infty L_\nu/h\nu\left[\int_0^\infty e^{-\tau_\nu} n_{\rm H^0}\sigma_\nu dr\right]d\nu = \int_0^\infty n_p n_e \alpha_{\rm B,H}(4\pi r^2)dr$$

The square bracket term in the left side is

$$\int_0^\infty e^{-\tau_\nu} n_{\mathrm{H}^0} \sigma_\nu dr = \int_0^\infty e^{-\tau_\nu} d\tau_\nu = 1$$

Then, we obtain

total number of ionizing photons

$$Q_0 = \int_0^\infty n_p n_e \alpha_{\mathrm{B,H}} dV, \quad \text{where } Q_0 \equiv \int_{\nu_0}^\infty \frac{L_\nu}{h\nu} d\nu \text{ and } dV = 4\pi r^2 dr$$

- Assuming that ***the ionization is nearly complete (*** $n_p = n_e = n_{\mathrm{H}}$ ***) within*** $R_s$, and nearly zero ( $n_{\mathrm{H}^0} = n_{\mathrm{H}},\ n_e = 0$ ) outside $R_s$, we obtain the size of the ionized region.

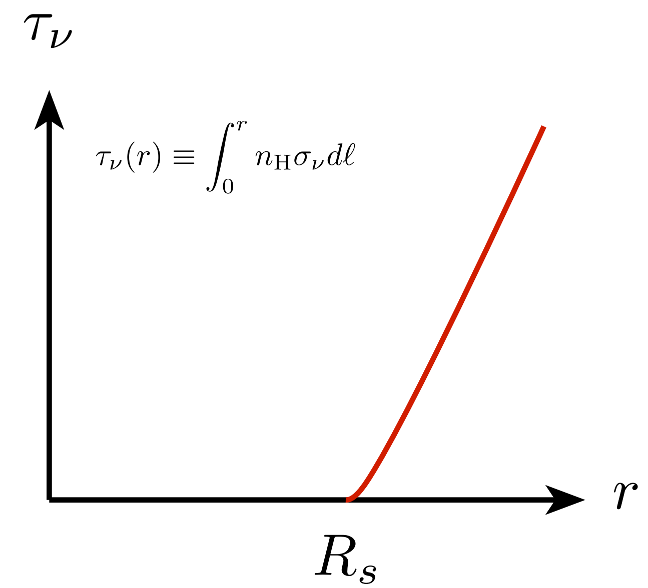
$$Q_0 = n_{\mathrm{H}}^2 \alpha_{\mathrm{B,H}} \frac{4\pi}{3} R_s^3$$

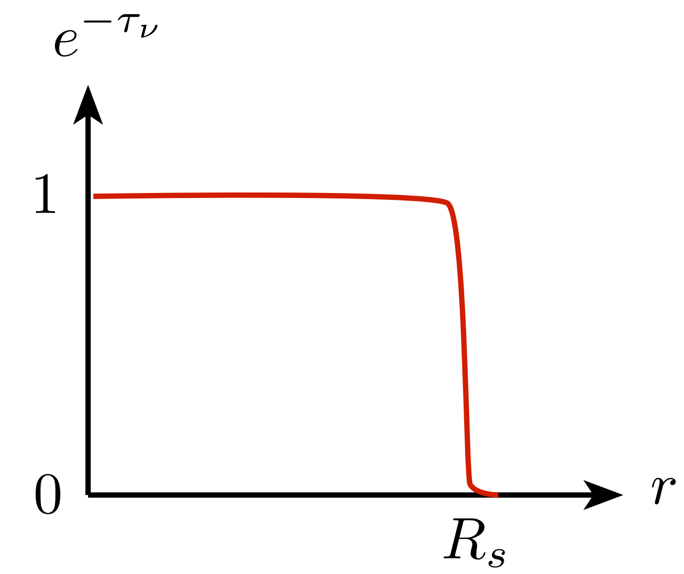
$$R_s = \left( \frac{3}{4\pi} \frac{Q_0}{\alpha_{\mathrm{B,H}} n_{\mathrm{H}}^2} \right)^{1/3}$$

$$= 3.17 \left( \frac{Q_0}{10^{49}\ \text{photons s}^{-1}} \right)^{1/3} \left( \frac{n_{\mathrm{H}}}{10^2\ \mathrm{cm}^{-3}} \right)^{-2/3} \left( \frac{T}{10^4\ \mathrm{K}} \right)^{0.28} \quad [\mathrm{pc}]$$

The physical meaning of this is that ***the total number of ionizing photons emitted by the star balances the total number of recombinations within the ionized volume*** $(4\pi/3)R_s^3$ , often called the Strömgren sphere. It's radius $R_s$ is called the Strömgren radius.

- ***Opacity as a function of distance***
  - We note that the medium is fully ionized within the Strömgren sphere. Thus, within the Strömgren sphere, the opacity is nearly zero. The opacity suddenly increases at the boundary of the ionized region.




- ***Mean free path***
  - The mean free path of an ionizing photon is

$$\lambda_{\rm mfp} = \frac{1}{n_{\rm H}\sigma_{\rm pi}} \sim 5 \times 10^{-4}\,{\rm pc} \left(\frac{n_{\rm H}}{10^2\,{\rm cm}^{-2}}\right)^{-1} \left(\frac{\sigma_{\rm pi}}{6.304 \times 10^{-18}\,{\rm cm}^{-2}}\right)^{-1}$$

This tells us that the transition from ionized gas to neutral gas at the boundary of the H II region will occur over a distance that is very small compared to the Strömgren radius.

- Time Scales:
  - ***Ionization time scale***: The Strömgren sphere analysis assumes a steady state solution. What is the time scale for approach to the steady state? Suppose that we start with a neutral region, and the ionizing source is suddenly turned on.

$$
\begin{aligned}
t_{\text{ioniz.}} &= \frac{\text{total number of ions to be created}}{\text{number of photons supplied per unit time}} \\
&= \frac{(4\pi/3)R_s^3 n_{\mathrm{H}}}{Q_0} = \frac{1}{\alpha_{\mathrm{B,H}} n_{\mathrm{H}}} = 1.22 \times 10^3 \left( \frac{10^2\,\mathrm{cm}^{-3}}{n_{\mathrm{H}}} \right) \quad [\mathrm{yr}]
\end{aligned}
$$

  - ***Recombination time scale***: Suppose that the ionizing source suddenly turns off. The ionized region will recombine on the recombination time scale:

$$
t_{\text{rec}} = \frac{1}{\alpha_{\mathrm{B,H}} n_{\mathrm{H}}} = 1.22 \times 10^3 \left( \frac{10^2\,\mathrm{cm}^{-3}}{n_{\mathrm{H}}} \right) \quad [\mathrm{yr}]
$$

Note that the recombination time scale is identical to the ionization time scale!

The ionization/recombination time scale is shorter than the main-sequence lifetime > 5 My for a massive star.

# Ionization of Helium

- Now, what about helium?
  - Out of every 1000 atoms, there are on average 912 hydrogen atoms, 87 helium atoms and one heavy atom.
    - Looking at the photoionization cross sections for $H^0$, $He^0$, $He^{+1}$, we see that above the 24.6 eV threshold for ionizing $He^0$, the photoionization cross section for helium is larger than that for hydrogen.

$$\begin{aligned}\sigma_{\mathrm{pi,He^0}} &\approx 6.5\,\sigma_{\mathrm{pi,H^0}} \quad \text{at } h\nu \sim 24.6\,\mathrm{eV} \\ &\approx 14\,\sigma_{\mathrm{pi,H^0}} \quad \text{at } h\nu \sim 54.5\,\mathrm{eV}\end{aligned}$$

    - Thus, the photoionization cross section for He is ~ 10 times that of H, while the number density of He is ~ 0.1 times that of H.
    - This implies that if we suddenly turn on a hot star, ***the initial photons in the range*** $24.6\,\mathrm{eV} < h\nu < 54.4\,\mathrm{eV}$ ***will be about as likely to photoionize a helium atom as a hydrogen atom.***
    - ***In the range of*** $13.6\,\mathrm{eV} < h\nu < 24.6\,\mathrm{eV}$***, on the other hand, nearly all the photons go to ionize H***; scarcer atoms (metals like O and C) account for only a tiny fraction of the ionizations.

- ***Radiative Recombination of Helium***

$$\mathrm{He}^{++} + e^- \rightarrow \mathrm{He}^+$$

$$\alpha_\mathrm{A}(T) \approx 4.13 \times 10^{-13}\, Z (T_4/Z^2)^{-0.7131-0.0115\ln(T_4/Z^2)} \quad [\mathrm{cm^3\,s^{-1}}] \quad (30\,\mathrm{K} < T/Z^2 < 3\times 10^4\,\mathrm{K})$$

$$\alpha_\mathrm{B}(T) \approx 2.54 \times 10^{-13}\, Z (T_4/Z^2)^{-0.8163-0.0208\ln(T_4/Z^2)} \quad [\mathrm{cm^3\,s^{-1}}]$$

$$\mathrm{He}^{+} + e^- \rightarrow \mathrm{He}^0$$

$$\alpha_{1\mathrm{s}^2,\mathrm{He}} = 1.54 \times 10^{-13}\, T_4^{-0.486} \quad [\mathrm{cm^3\,s^{-1}}] \quad (0.5 < T_4 < 2)$$

$$\alpha_{\mathrm{B},\mathrm{He}} = 2.72 \times 10^{-13}\, T_4^{-0.789} \quad [\mathrm{cm^3\,s^{-1}}]$$

Here, $\alpha_{1\mathrm{s}^2,\mathrm{He}}$ is the recombination rate to the ground state $1\mathrm{s}^2\,{}^1S_0$.,

and $\alpha_{\mathrm{B},\mathrm{He}}$ is the recombination rate coefficient to all states except the ground state.

Note: $\alpha_{\mathrm{B,H}} \approx \alpha_{\mathrm{B,He}}$ and $\alpha_{\mathrm{A,H}} \approx \alpha_{\mathrm{A,He}}$.

# Homework (due date: 05/08)

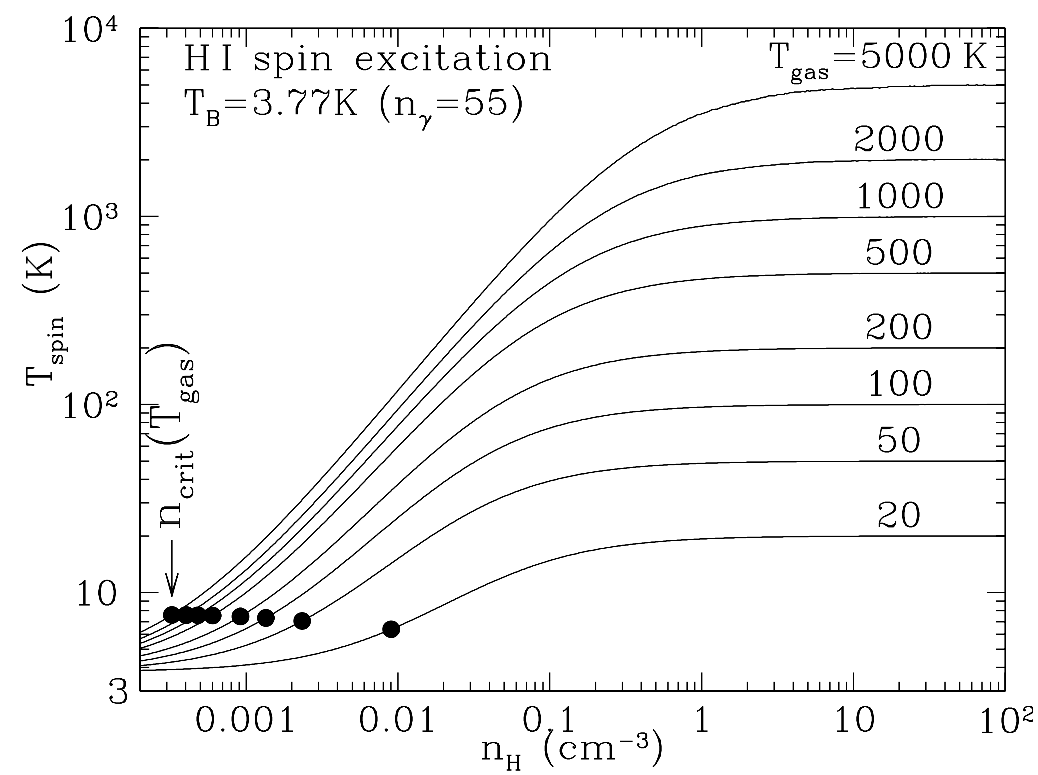
[Q9]

(1) If we consider only the background radiation field and collisions with hydrogen, the spin temperature of the 21-cm transition is given by

Eq(a): $$T_{\rm spin} = \frac{T_{\rm rad} + y_c T_{\rm gas}}{1 + y_c} \quad \text{where} \quad y_c \equiv \frac{T_*}{T_{\rm gas}} \frac{n_c k_{10}}{A_{10}}$$

- Using the above equation, make a plot similar to the right side figure. (Extrapolate the approximate formula for $k_{10}$ down below 20 K and up above $10^3$ K.)
- Denote the two critical densities, for each gas temperature, defined by

Eq(b): $$n^*_{\rm crit} = \frac{T_{\rm gas}}{T_*} \frac{A_{10}}{k_{10}} \quad \text{and} \quad n_{\rm crit} = \frac{(1 + n_\gamma) A_{10}}{k_{10}}$$




(2) Discuss whether Eq(a) for the spin temperature for the 21-cm transition can be applied to the [C II] 158μm line or not.

Explain why the equation cannot be applied?

- [Q10]
  - Define $E_x$ to be the energy at which the photoionization cross section for a hydrogenic ion is equal to the Thomson scattering cross section:

$$\sigma_T = (8\pi/3)(e^2/m_e c^2)^2 = (8\pi/3)(\alpha^2 a_0)^2$$

(a) Express $E_x/I_{\rm H}$ in terms of $Z$ and the fine structure constant $\alpha \equiv e^2/\hbar c = 1/137.04$

(b) For hydrogen, calculate $E_x$ in eV.

Hint: use the cross section formula for high energies.

- [Q11]
  - The Einstein A coefficients for all the allowed transitions of hydrogen from levels $n \leq 3$ are given in the table below:

| $u$ | $\ell$ | $A_{u\ell}(\mathrm{s}^{-1})$ | $\lambda_{u\ell}(\text{Å})$ | |
|---|---|---|---|---|
| $3d$ | $2p$ | $6.465\times10^7$ | 6564.6 | H$\alpha$ |
| $3p$ | $2s$ | $2.245\times10^7$ | 6564.6 | H$\alpha$ |
| $3s$ | $2p$ | $6.313\times10^6$ | 6564.6 | H$\alpha$ |
| $3p$ | $1s$ | $1.672\times10^8$ | 1025.7 | Ly$\beta$ |
| $2p$ | $1s$ | $6.265\times10^8$ | 1215.7 | Ly$\alpha$ |

(a) Consider a hydrogen atom in the $3p$ state as the result of radiative recombination:

$$p + e^- \;\rightarrow\; H(3p)$$

What is the probability $p_\beta$ that this atom will emit a Lyman β photon?

(b) In an H II region where hydrogen is the only important opacity source, what is the mean number of times a Lyman β photon, produced as the result of $p + e^- \to H(3p)$, is "scattered" (that is, absorbed and then re-emitted) before an Hα photon is emitted?

Hint: you may want to use the following formula:

$$\sum_{n=1}^{\infty} nq^n = q\sum_{n=1}^{\infty} nq^{n-1} = q\frac{d}{dq}\sum_{n=1}^{\infty} q^n = q\frac{d}{dq}\left[\frac{q}{1-q}\right] = \frac{q}{(1-q)^2}$$

- [Q12]
  - Absorption line observations of an interstellar cloud measure column densities:

$$N(\mathrm{CaI}) = 1.00 \times 10^{12}\,\mathrm{cm}^{-2}$$
$$N(\mathrm{CaII}) = 3.08 \times 10^{14}\,\mathrm{cm}^{-2}$$

The gas temperature is estimated to be $T = 50$ K. At this temperature the radiative recombination coefficient for $\mathrm{Ca\,II} + e^- \to \mathrm{Ca\,I} + h\nu$ is $\alpha = 1.3 \times 10^{-11}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$. The starlight within the cloud can photoionize $\mathrm{Ca\,I} + h\nu \to \mathrm{Ca\,II} + e^-$ with a photoionization rate $\zeta = 1.2 \times 10^{-10}\,\mathrm{s}^{-1}$. Estimate the electron density $n_e$ in the cloud.